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HOW THE STOCHASTIC PROBLEM DRIVES THE BRAZILIAN ELECTRICITY SECTOR

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Abstract: The Brazilian Electricity Sector (BES for short) is the ninth biggest in the world in terms of electrical energy generation. In 2011, the generation, distribution and transmission sectors had together (an income statement) a revenue of approximately USD 8.3 billion; currently, electricity reaches more than 99% of the Brazilian houses. Brazil has approximately 90% of its energy supplied by hydroelectric plants. One of the main characteristics of the generation systems that have hydraulic predominance is the strong dependence on the hydrological regimes. Thus, this paper aims to show that the stochasticity verified in the uncertainty related to the affluences is associated to the three fundamental activities of BES: expansion planning, operation planning and energy Spot Price. In other words, the energy synthetic series are crucial to determine which is the best way to operate the sector, providing support to decisions on when it should, or not, happen an expansion, avoiding, unnecessary costs and/or losses to the system. Finally, the paper presents the periodic autoregressive model, a particular structure of the Box & Jenkins family, denoted by PAR (p), employed to model the series of hydrological streamflow used for estimating the operational costs of the Brazilian hydro-thermal optimal dispatch.

Key words: Brazilian Electricity Sector, stochasticity, operation planning, expansion planning; energy spot price.

HOW THE STOCHASTIC PROBLEM DRIVES THE BRAZILIAN ELECTRICITY SECTOR

1. Introduction

Brazilian Electricity Sector (BES) is the ninth biggest in the world in terms of energetic generation, producing approximately 470 TWh(OECD, 2012). In 2011, the generation, distribution and transmission sectors had together a revenueof about USD 8.3 billion (ABINEE, 2012); currently, electricity reaches more than 99% of the Brazilian houses.

To the quadrennial 2011-2014, it is estimated that the investments in the Electricity sector will reach USD 56 billion (PAC, 2012), the second largest in relation to the investments among the infrastructure sectors. The National Development Bank (*BancoNacional de Desenvolvimento*, BNDES) is planning to invest about USD 34 billion in the sector, which is 10% of its whole investment in this quadrennial(BNDES, 2012).

Besides the considerable economical wealthy of the BES, due to a historical use of the natural resources for the electrical energy generation, it was also developed the interconnection of the national grid (SIN for short). The continental size of the SIN allows that the energy generated at any region, may the consumed by users at different regions. By interconnecting the regions, the SIN allows a better use of the natural resources. As a consequence, the resulting system (BES) has a huge dimension, very complex and requires computational tools to give support on the operational planning of the SIN.

Then, in this context, Brazil has a system that involves billions of dollarsin generation, distribution and transmission. This system is eminently hydraulic, nationally interconnected, and mixed in terms of investments (that is, it combines public and private companies); and it also has, in terms of market structure, monopolies, mainly in the transmission area, and big oligopolies, in the areas of generation, distribution and transmission. This situation brings up a series of problems and challenges that require processes of decision making in different time horizons, long term (5 to 10 years – probability of future energy storage, expected value of the future costs of the thermal generation etc), medium term (5 to 1 year – yearly contracts for the peak hours expected demand etc.) and short term (up to 1 year – control of the energy flow, hourly dispatch etc).

Such problems have different dimensions and are related to decision-makings directly linked to the three pillars that support the BES: expansion planning; operation planning; and accounting and settlement process of the energy transactions in the short term market. These functions are developed by the Energy Research Center (*Empresa de PesquisaEnergética*, EPE), Electric System National Operator (*OperadorNacional do Sistema*, ONS) and Board of Trade of Electricity Energy (*Câmara de Comercialização de EnergiaElétrica*, CCEE).

From this overview, some questions arise naturally: how much energy should be generated considering the hydraulic and the thermal possibilities? Which is the right moment to save water and to use fossil fuels? Should it be an investment towards enlarging the system’s capacity, or is it time to save and wait for a future moment with a bigger expansion? Which should be the short term energy price, in order to pay all the production factors and to observe the tariff moderateness?

These are some of the daily questions in the Brazilian Electricity Sector and the answers to them are far from trivial. They all involve a comprehensive planning and a synchronized management. Nevertheless, there is a common feature that connects the electricity sector’s three pillars which is the stochasticity associated to the expansion planning, operation planning and Energy spot price.

In order to carry out the expansion and operational planning and estimate the short term energy price, BES opted for the use of a chain of computer modules, that has the property that as the planning lead time reduces, the uncertainty representations also reduce, while the representation of the physical characteristics increases (Maceira*et. al.*, 2012). The highest planning horizon module of this chain is known as NEWAVE(CEPEL, 2011a). In this program, the optimal strategy, i.e., the optimal share between the hydro and the thermal generation is obtained by a stochastic optimization procedure that uses the Stochastic Dual Dynamic Programming (SDDP), which determines the future costs function (FCF) that is used in the coupling process of the other modules of the chain of computer programs;

Such optimization process has as the unique stochastic process the natural inflow of energy[i] (NIE, for short), which is artificially generated by a Periodic Autoregressive model (PAR(p) for short), whose hyperparameters are adjusted by the available historical values. One important point must be emphasized; in order to avoid undesirable negative values for the simulated NIE, the PAR(p) adopted within the BES structure, assumed a Log-Normal distribution for the model residuals in the Monte Carlo generation of the artificial NIE series.

The expansion and operational planning used in the BES has been questioned in the latest years, due to its lack of efficiency in the correct representation of the scenarios produced by the simulated NIE series, producing distortions on the energy prices, especially on the difference liquidity prices (DLP for short), which is used by the official bodies to compensate on price differences at the moment of the energy trades. As an example of that, in June 2010 the Brazilian regulator (ANEEL) put forward a determination (determination # 2654 of 24/06/2010) whereby the forecasts of the inflows produced by the Monthly Operational Program (MOP) should be reviewed, as, in some weeks, the estimated DLP presented substantial increases and, in others, substantial decreases.

As one can see and it will be reinforced throughout the article, the evidence of the stochasticity present in the three pillars of the BES is unquestionable, it is clear that the crucial element in this context are the generation of the simulated NIE scenarios, produced by PAR(p) structures (one for each month of the year). From them, the spot prices are generated and the decisions on the operation and expansion policies of the sector are taken.

With these ideas in mind, the main task of this article is to clarify the relationship: *"stochasticity" vs"brazilian electricity sector"*, by showing the stochastic model adopted and raise methodological questions that are peculiar to the BES. Such questions may explain the problem mentioned above concerning the inconsistency of the energy spot price.

To achieve this goal, the remaining paper is divided in three sections following this introduction. Section 2, entitled "The relationship between the stochasticity and the BES" brings up thedetails of the sector, identifying the relations among the three fundamental pillars mentioned and the stochasticity. It is followed by section 3, where the PAR(p) model, which is the stochastic model adopted by BES is described and discussed. Finally, the last section (section 4) is dedicated to some concluding remarks.

2. The relationship betweenStochasticity and the Brazilian Electricity Sector

During the nineties and the beginning of the years 2000, some important reforms were carried out in the Brazilian Electricity Sector ((Fernandes*et. al.*, 2005); (Carpio & Pereira Jr, 2006)). After the 2003 reform, the Central Government defined a complex and consistent set of official agents with well defined tasks and competences, aiming three basic objectives: cheap tariffs, reliable energy supply system and universality of the distribution of energy(Tolmasquim, 2011).

Following these reforms, the energy sector agents can be classified into three levels, following their juridical nature and their institutional competences. They are; those that execute government activities; those that perform regulatory activities, and those rerlated to private law entities that run special activities.

Government activities are conducted by the National Council for Energy Policy (*ConselhoNacional de PolíticaEnergética*, CNPE), the Ministry of Mines and Energy (*Ministério de Minas e Energia,*MME) and by the Electricity Sector Monitoring Committee (*Comitê de Monitoramento do SetorElétrico*, CMSE). The regulation activities are conducted by the Brazilian Electricity Regulatory Agency (*AgênciaNacional de EnergiaElétrica*, ANEEL). Private entities conduct the technical activities: medium and long term sectoral planning(EPE), feasibility of trading activities (CCEE) and National Interconnected System coordination (ONS). Figure (1) presents the relationship among the institutional agents.Asalready highlightedin this paperanddiscussedin the following sections, the stochasticitydirectly interferesin the activities ofEPE, ONSandCCEEthrough the generation of synthetic series of NIE.

The next step of this study is to show how operational entities, with different purposes, are connected to the stochasticity associated with NEWAVE module, which supports the expansion planning (EPE), the operation of the system (ONS) and the price of short-term energy (CCEE).

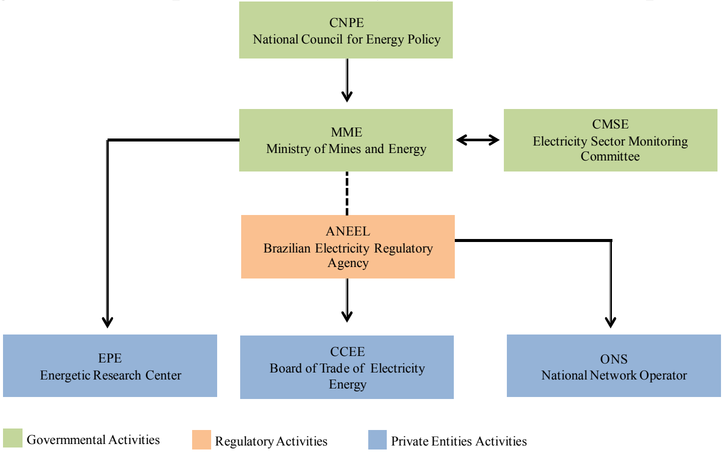

Figure 1– Electrical Sector's Institutional Agents

To achieve this objective in a clear and consistent manner, it is first presented the characteristics of the Brazilian Electricity Sector addressing issues related to implementation of the three activities mentioned above, followed by discussing separately such activities, showing how stochasticity, linked to NEWAVE model, helps in implementing them.

The BES presents some peculiar characteristics that make it different from any other in the international context, and this makes the system's operation and planning a complex activity with complex implementation. According(Marreco, 2007), both the technical and the economic aspects must be highlighted. The energy sector has specific characteristics in relation to other sectors, such as the natural monopoly (electric sector as a whole) and network industry (for example, distribution and transmission segments). In addition to the features described, there are some others specificities in the BES, such as: (*i*) hydroelectric base, with cascading plants of different ownerships; (*ii*) high capital intensity; (*iii*) long term investment maturation; (*iv*) huge interconnections; and (*v*) considerable uncertainties (hydrological, demand growth, fuels price, among others).

Concerning the high capital intensity and the long term investment maturation, it is emphasized that the time between the decision to build a hydroelectric plant and its effective entry into operation may be greater than ten years. Another feature is the sunk costs, due to the high degree of specificity of the activities associated with the sector (*e.g.* electrical energy transmission lines). This scenario emphasizes the importance of the long term planning conducted by EPE.

Another technical attribute fundamental to the sector is the physical balance, which requires the system coordination, since its parts work with strong interdependence. The electrical energy generation can be obtained through various technologies with different costs and socio-environmental impacts. In the case of the hydroelectric plants, for example, the input is a random flow based on the rainfall, which implies a great complexity in the short and medium term planning process, a task performed by ONS.

Even with an efficient electrical energy demand and supply planning system, due to the magnitude of the ventures and the market peculiarities, there is a short term energy market whose final objective is to solve the differences between supply and demand, operated by CCEE.

Seeking to facilitate the system's operation and planning, the Electrical sector agents created the National Interconnected System (SIN)(Figure2).This interconnection comes from the fact that the country adopts a system that is basically hydro and the water reservoirs of the facilities are used in a planned way, so that it is possible to profit from the rainfall diversity in the different existent basins; this procedure assures the Brazilian system an important energetic gain. Such SIN, of continental dimensions, allows, the energy generated at any point in the country be consumed by different consumers in different regions (Carpio & Pereira Jr, 2006). The interconnection between regions results in better use of the resources.

Within this context, the coordination of the operation of the SIN by ONS, according to general guidelines of the Monitoring Committee of Energy, aims to meet the requirements of power consumption of the system in order to guarantee the continuity of the supply of energy and reduce operating costs. Thus, the rational use of resources must be planned aiming to meet the requirements of the present system and seek ways to determine the needs for system expansion (EPE) with subsequent investments.

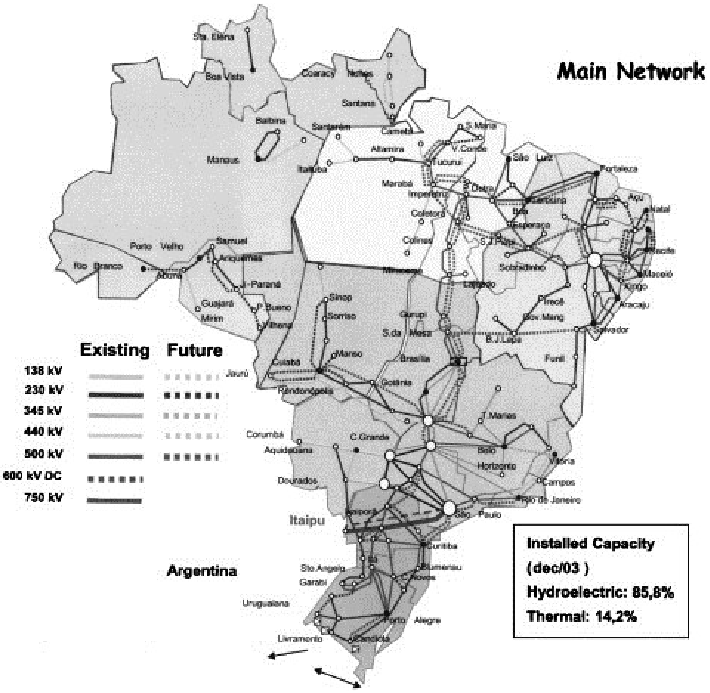

Figure2– NationalInterconnected System

The considerations that underline the specificities of the electricity sector in relation to expansion planning, operation planning and valuation of the spot price of energy highlight the prominence of the theme. These decisions are often made before a climate of uncertainty and require systematic processes for decision support, especially about future prospects.

To confirm what has been discussed so far, the next three sections present the planning of expansion, the planning of the operation, and the determination of the spot price of energy, separately identifying characteristics peculiar to each segment, and the relationship with the stochasticity inherent in the sector

For ease of exposition of the arguments previously portrayed, it is first considered the planning of the operation of the system, followed by the expansion planning and concluding with the determination of the spot price. It is shown how way the stochastic component affect each one of them.

2.1. The Operation Planning of the BES

The system ofelectrical power generationin Brazil, withapproximately 105,000MW, is basicallyhydropower. The shareof this energy sourceis approximately80,000 MW, representing approximately 70% of theinstalled generation capacity(and, under normal conditions, isresponsible for producingapproximately 90% of the electricity consumedin the country). In turn, thethermoelectric and thethermonuclearplants represent 18% and 2% of generation capacity, respectively(ANEEL, 2012). To have anidea of theimportance of thishydro system, for example, 78% of China's output of electrical power is thermal, and out of this more than 90% is coal(Andrews-Speed & Dow, 2000).

In this context, a hydrothermal system in which one has, at one hand, the uncertainty on the future availability of hydro energy, at a cost of generation considered zero, and on the other hand, the thermal energy with high cost of generation, emphasizes the need to decide at every moment what is the best share of hydro and thermal plants.

In the operationofhydrothermal systems, unlike whathappensinpurelythermalsystemswhere theoperationplanning problemcan be solved by finding acombination ofplantsthat minimizefuel costs, in the hydrothermal systems, the decision makingis coupledin time, *i.e.*a decisiontaken todaywill haveconsequencesin the future (Terry *et. al.*, 1986).

For example, if there is asignificanthydroelectricorderbefore adry period, one may risk the use ofthermaldispatchathigher costin the future. On the other hand, athermal dispatchbefore awet periodmay causeloss, incurring wastedenergy (Figure3).

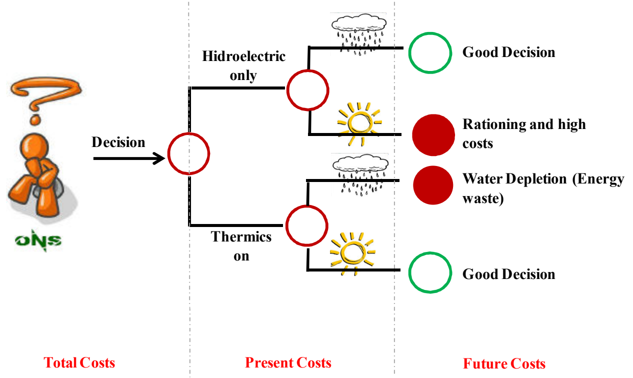

Figure3–Problem of Operation Planning Decision

The optimal system operation will involve, therefore, a compromise between using or not using water from the reservoirs. The decision variable is the water volume stocked at the end of the period of operation (end volume).

Such a decision has an immediate cost, associated to the thermal generation (ICF - immediate cost function), and a future cost, associated to the expectation of thermal dispatch, indicated by the future cost function (FCF). The total cost is the sum of these costs and the optimal decision is obtained equating to zero the derivative of the total cost in relation to the end volume equal to zero (Pereira et. al, 1998).

As it is possible to observe in (Figure 4),the ICF increase with the growth of the final volume; that is, the higher this water volume is, the bigger is the spent with thermal generation. The FCF has an opposite behavior: as the final stock grows, the future expectation with fuels expenditures diminishes.

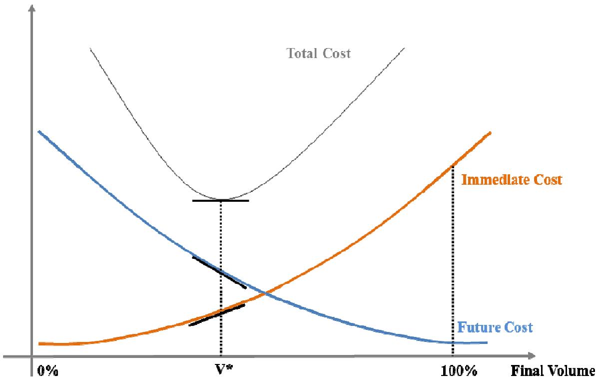

Figure 4 – Operation Planning Criterion

This operation criterion, although apparently simple, involves complex calculation, causing the operation of a coordinated hydrothermal system to be made in different stages, through the chain models that enables the definition of an operation strategy.

As highlighted by (Pereira & Pinto, 1982) and (Lepecki & Kelman, 1985), the coordination of the planning of the operation of a hydrothermal system, such as the Brazilian SIN, can be formulated as large scale optimization problem, coupled with temporal and spatial operation, dynamic, stochastic and nonlinear. Its solution requires that the problem is decomposed into a chain of coupled models that consider long-term horizons (probability of future energy storage, expected future value of thermal generation etc.), medium term (annual contracts to guarantee peak hours supply etc) and short-term (flow control, dispatch time).

To proposed solution to this problem adopted by the BES is the use of a chain of coupled models that consider different planning horizons: long, medium, short terms and daily programming (Maceira*et. al.*, 2002). The coupling among the models is made through the future cost function of the optimal energy operation.

More specifically, it is defined as operation planning, efforts to shape the behavior of the system at a horizon of five years, promoting the rational use of resources to ensure quality and safety in meeting the market demand. This is carried out by minimizing the operating costs of the hydrothermal system.

The operation programming is to establish a short-term procedure of the hydrothermal system providing operational decisions of the generation system that are feasible to the transmission system and respect the goals set in the planning of the operation.

To solvethe models adoptedin the chain ofBES'senergy planning, it is used the StochasticDualDynamic Programming(SDDP) ((Shapiro *et. al.*, 2011); (Pereira & Pinto, 1991); (Pereira & Pinto, 1985); (Pereira, 1989)).This methodologyuses the techniqueofBendersdecomposition ((Benders, 1962); (Pereira & Pinto, 1983)) seeking to findoptimal strategiesfor the operation ofinterconnectedsubsystems, whileinflowsare treated as a periodicautoregressive model(Hipel & McLeod, 1994).

The SSDP is used to determine the operating policy that minimizes the expected value of the expected cost of the operation to the horizon of up to five years. The strategy is checked through a simulation process that makes a thorough use of a series of natural energy, representative of each subsystem. Such series may be either historical data or synthetic data. The set of hydroelectric plants is represented by equivalent reservoirs in which a number of hydroelectric plants is aggregated into a single equivalent reservoir that receives, stores and discharges energy (Natural Inflow Energy, NIE) instead of water. On the other hand, the thermal plants are represented through their operation costs of minimum and maximum generation. The NEWAVE models produce future cost function (FCF) that is attached to the short-term module at the end of planning horizon (Figure 5).

In order to reducethe computational effortrequired by theoptimization procedure, the models usedfor planning theoperationin the medium termrequirethe aggregation ofplantsintoequivalent reservoirs ofenergy for each subsystem(Southeast/Midwest, South, Northand Northeast). This way, theSINisrepresented by four interconnected subsystem, each one with its equivalent fictions reservoirs.

However, in order to check if the operating policy obtained by the strategic decision model is viable, it is necessary to disaggregate the solution of the fictitious equivalent reservoirs into individual plants, *i.e.* check whether the plants that make up the equivalent system will be able to meet the amount of hydroelectricity for the system, given by the model of strategic decision. For this purpose, the DECOMP and the DESSEM models are used to produce the weekly and the daily programme, respectively (see figure 5), departuring from the optimal decision obtained from the NEWAVE system.

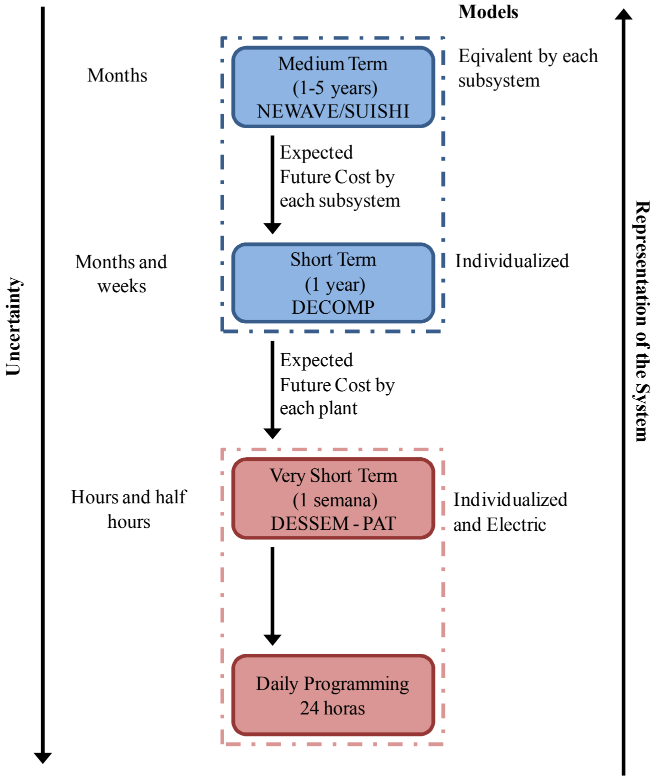

Figure 5 - Chain of Models in Operation in the Brazilian Electric Sector

As already stated, this article will focus on the stochastic module that supports the planning model NEWAVE. As noted, such a model is used, in general, to optimize the use of water and, therefore, depends on the rainfall stochasticity, which is represented by synthetic series generated from time series models that are included in Inflow Energy Modules (IEM) (Figure 6).

In the short term, the aspects of quality and service demands consist of meeting the generation targets set in medium-term planning and monitoring operational feasibility in terms of restrictions of generation and transmission equipment. In this planning horizon, the DECOMP model is used and its goal is the minimization of the expected value of the total system cost.

When the study horizon is reduced to the short and very short term (DECOMP and DESSEM-PAT), the representation of the system is refined to hydroelectric plants, expressing their operating characteristics of water and energy restrictions. In operation programming the horizon is up to two hours with discretization into hours or thirty minutes (Figure 5). The goal is to obtain the optimal dispatch of the hydrothermal system, addressing the aspects related to energy, hydrology and electrical. The horizon of short and very short term (daily schedule) allows one to consider the values of inflows known or deterministic (Maceira*et. al.*, 2002).

Finally, it is important to notice that as the time interval decreases, the uncertainty regarding inflows/NIE decreases, reaching a deterministic extrapolation, and the level of details of the system representation increases as its features are considered individualized.

Following the logic proposed at the beginning of this section, it is presented the NEWAVE model, emphasizing the connection between this model and the model for generating synthetic series (Natural Energy Module), and DECOMP model, important for determining the short-term energy price and which is coupled to the model NEWAVE future cost function.

NEWAVE Model

The NEWAVE is an optimization procedure to model to the medium term planning (up to 5 years ahead) with monthly discretization and representation to equivalent systems. Its objective is to determine the hydraulic and thermal generation strategy of each stage that minimizes the expected value of the operation cost to the whole planning period.

One of the main outcomes of this module are the future cost functions, essential to determine the “water cost” and, consequently, the impacts of using the water stored in the reservoirs. In this model, the load and the deficit cost function can be represented in levels[ii]and this enables the consideration of interconnection limits among the subsystems (CCEE, 2012).

The NEWAVE is composed of 4 operational modules: (*i*) Equivalent System Calculation Module – calculates the equivalent energy subsystems, that is, based on the NIE time series and on the characteristics of them, it calculates the Affluent Natural Energy; (*ii*) Inflow Energy Module – estimates the stochastic model parameters and generates affluent energy synthetic series, using them to calculate hydrothermal operation policy; (*iii*) Hydrothermal Operation Policy Calculation Module – determines the most economic operation policy to the equivalent subsystems, considering the uncertainties in the future affluences, the unavailability of the equipments and the demand levels; (*iv*) Operation Simulation Module – simulates the system operation during the planning period for different scenarios of hydrological sequences (CEPEL, 2011a).

As can be observed in (Figure 6), the four modules are interconnected. It is possible to observe that the Affluent Energies Module receives NIE’s series from the equivalent system calculation module, estimates the PAR(*p*) model and generates synthetic series to each subsystem, and provides the simulation and the operation policy modules with the parameters generated by the model and with the energy synthetic series. With these stochastic variables, the simulation and the operation policy modules determine the optimal operation policy using the SDDP tool.

Given that the market forecasting is defined by the agents and is fixed, it is clear that the only stochastic variable used in the optimization of the Brazilian hydrothermal system are the energy synthetic series generated by the affluent energies stochastic module. In other words, the scenarios provided by the Inflow Energy Module, that encompasses stages (*i*) e (*ii*) of the NEWAVE module, will ultimately strongly influence the three basic pillars of the Brazilian Electric Sector: the operation, the planning and the energy price settlement.

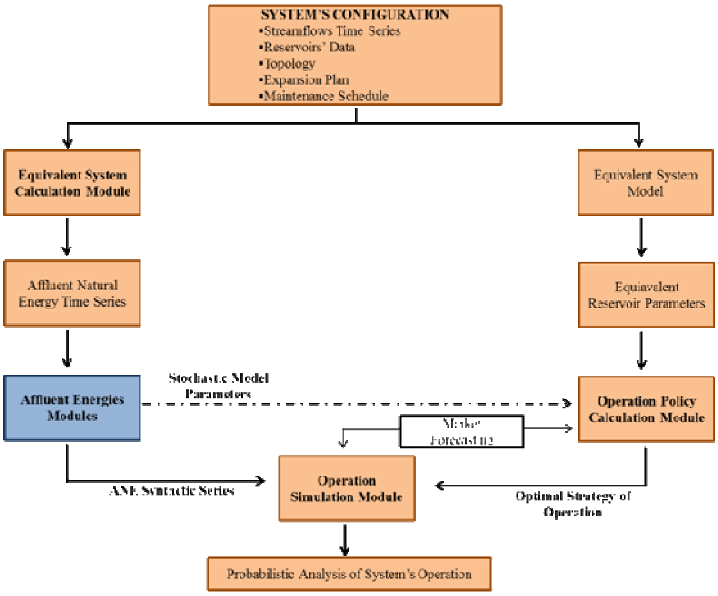

Figure 6 - Relation among the NEWAVE Modules

DECOMP Model

Like NEWAVE, the DECOMP model also seeks for an optimal operation of the hydrothermal system, but in a shorter time horizon (CEPEL, 2011b). The DECOMP solves the problem of planning in the short term operation of a hydrothermal system. As shown in (Figure 7), using the future cost function obtained in running the NEWAVE package and information on the load, inflows, availability and transmission limits among the submarkets, the module DECOMP produces the result for the optimized planning of the first month on a weekly base. Its main features are the short-term planning with weekly discretization in the first month of the study.

It is important to notice that the predicted inflows and randomness of the remaining inflows for the rest of the period are obtained through a tree of possibilities and a individual generating plant (as opposed to the aggregate format used in NEWAVE). As noted, the DECOMP module is used for short-term operation of BES and, as with NEWAVE, depends strongly on the stochastic models, *i.e.*, the stochasticity influences indirectly the DECOMP through the FCF and directly on the tree of inflow scenarios.

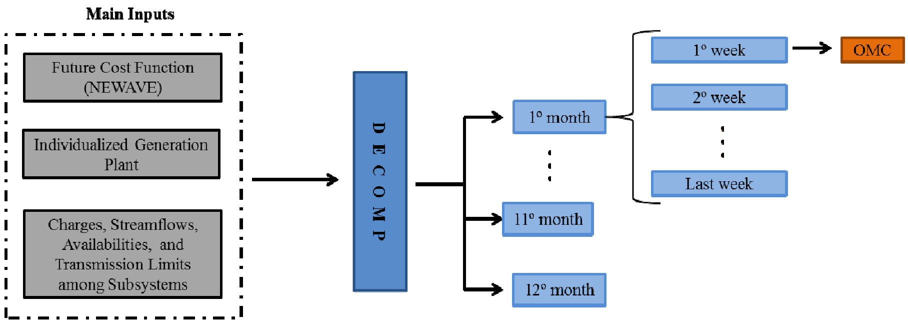

Figure 7 - DECOMP Program Main Inputs and Outputs

2.2. Expansion Planning of the BES

According to (Tolmasquim, 2011), just like the operation planning involves the compromise between the immediate use and the future use of water, in the case of the expansion planning, there is the relation between the future and the immediate use of the financial capital available to the expansion.

To put it differently, the agents need to decide between investing now and run the risk of system'sexcess supply due to the demand growth below the expected, or postpone the investment and run the risk of rationing, as can be seen in(Figure 8).

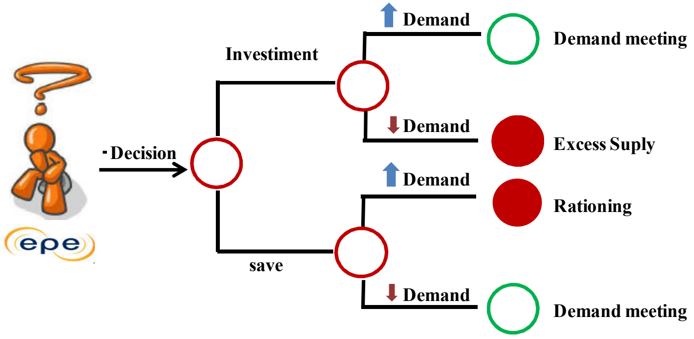

Figure 8 - Investment Decision

Following the same logic of the water use in the operation, the decision to expand the system capacity diminishes the stock of capital, with an immediate cost (expansion cost, which is known) and a future cost (deficit cost, which is estimated).

This way, the decision to "save" has low immediate cost and high future cost, the latter due to the increase in fuels consumption and rationings. On the other hand, the decision to "invest" has high immediate cost and low future cost (Figure 9) (Tolmasquim, 2011).

As in the operation process, in the planning case, for a given predicted demand, there are various possibilities for the expansion. Each possibility is equivalent to a level of reliability R (x - axis), where the expansion costs will depend on the demand, which is unique and the level of reliability R desired by the planning responsible.

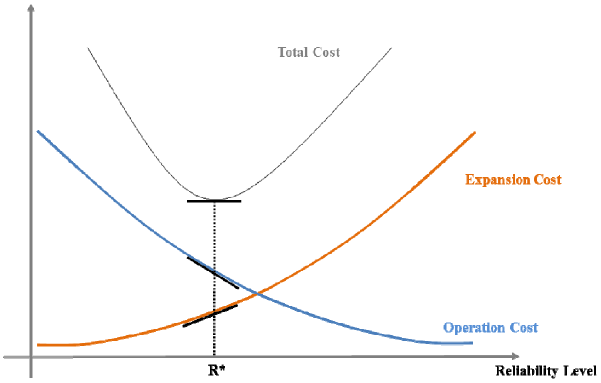

Figure 9 - Expansion Planning Criterion

Similarly to what happens with the operationprocess, the solution to the planning process is given by an optimization exercise, having as decision variable the reliability level (R) and, as objective function, the total cost (sum of expansion and operation costs). It is worth remembering that the reliability level (R*) and expansion plan are optimal in the minimal total cost point (Tolmasquim, 2011).

In the optimal plan, the derivative of the expansion cost in relation to the demand represents the Expansion Marginal Cost (EMC), which is linked to the cost of an incremental load, with capacity expansion. While the operation cost derivative in relation to the demand represents the Operational Marginal Cost (OMC), being the cost to meet the incremental demand, without capacity expansion.

Considering that the demand increases in time, the Brazilian Electricity Sector expansion takes place when the OMC is equal to the EMC; in expanding, the OMC decreases in relation to the EMC and another cycle starts.

As in the planning of the operation, the expansion planning uses the NEWAVE model to support decision making. The calculation of OMC is performed by simulations of the operation of the system, based on the NEWAVE model. Since the calculation of the EMC is estimated based on the results of new energy auctions, considering that the winning bid of the more expensive project of the auction, which reveals the agents' willingness to invest, is a good approximation of the EMC.

Finally, it is crucial to observe that, given the time to the maturation of the investments in the electrical sector and the serious consequences of a rationing, the generation planning needs to meet the reliability level defined by the Energetic Policy National Council, through which the annual risk of deficit should not overcome 5% in any subsystem. Yet, to meet the economic criterion, it is necessary have equality between the OMC and the EMC.

In summary, it was verified that the stochasticity is intrinsically linked to the BES expansion planning, since, as observed, the decision to expand the system is intimately connected to the OMC, which is calculated from the NEWAVE model and has as one of the stochastic variable the energy synthetic series.

2.3. Electrical Energy Trading of BES

The New Model divided Brazilian energy market into twotrading environments: the Regulated Trading Environment (*Ambiente de ContrataçãoRegulada*, ACR), aiming at meeting the demands of the captive consumers, represented by term contracts on energy from the market *Pool*; and the Free Trading Environment (*Ambiente de ContrataçãoLivre*, ACL), dedicated to companies with bigger consumption volume, and where the bilateral contracts are freely negotiated, following specific trading rules and procedures (Castro & Leite, 2010).

However, due to the fact that the electrical energy physical attribute requires an instantaneous balance between demand and supply, the supply predicted *ex ante* is not necessarily equal to the observed demand; this calls for an instantaneous balance in two points: energy supply and financial accounting (Rodrigues, 2007).

In what regards the supply, the Electric System National Operator (ONS) centralizes the dispatch of the plants through the aggregation of generation and transmission ventures, so that it requires a more effective management of the energy production cost. In this case, it can happen that a generator, even with all its capacity contracted, is not able to offer this capacity to the system due to decisions of the operator.

Concerns the first, ONS, responsible for planning the operation of the system, centralizes the dispatch of the plants through the aggregation of generation plants and the transmission lines in order to perform a more effective management use of resources, and consequently, minimize the cost of energy. In this case, a generator, even with all its energy being contracted, may not have to supply it to the system due to the decisions of the operator.

The second is a function of theBoard of Trade of Electricity Energy (CCEE), which accounts the differences between what was produced or consumed and what was contracted. The positive or the negative differences are liquidated in the short term market (spot) and evaluated by the Differences Liquidation Price (DLP), which is weekly set to each load level and to each submarket, and having as its base the subsystem's OMC. The DLP is limited by a minimum and maximum price.

As observed, the price in this market does not follow the economic relation between supply and demand set by the agents, but is determined by a set of computational models (*e.g.*, NEWAVE, DECOMP), operated by the ONS and CCEE. The expectations in relation to the future electricity consumption and to the future ENA's regime play a determinant role in the use of the energy accumulated in the hydroelectric reservoirs, consequently on the short term energy price. Therefore, the expected minimum cost in a given horizon should take into account different inflow scenarios that result in different operational decision.

In short, in discussing the three pillars of BES, it was shown that the NEWAVE model calculates the optimal operating system policy, considering present and future costs. The predominance of hydroelectric generating facilities in Brazil makes the question of randomness of inflows an important problem in optimization of the total cost of operation in the time horizon considered, as the future cost is a function of future inflows that are random, while the immediate cost is a function of the current dispatch of hydro and thermal plants, the latter considered zero. It should be noted, however, that this marginal cost will be zero (corresponding to the cost of the water*, i.e.* the hydraulic generation) only in the situations of full reservoirs in the present and in the future, *i.e.*, arising from inflow series with very favorable hydrology.

Thus, it is quite clear that there is a strong relationship in the BES between the stochasticity and the three pillars of the sector, *i.e.*, the synthetic series of energy/inflows are crucial in determining what is the best way to operate the sector, give support to decisions on when should the system be expanded or not, thus avoiding cost and/or unnecessary losses. And yet, they are an important factor in determining the short-term price of electricity, since the amount simulated/predicted of water at the reservoirs in the future will be one of the most important factor to set the short term price.

The next section discusses the stochastic Periodic Autoregressive model (PAR (p)) adopted by the Brazilian Electricity Sector to generate synthetic series of ENA, which are used in the NEWAVEmodel.

3. The stochastic model adopted: PAR(p) Model

According to (Hipel & McLeod, 1994), some historic series, among them the seasonal hydrological series show an autocorrelation structure that depends not only on the interval between the observations, but also on the period observed. Also according to (Salas & Obeysekera, 1982), stochastic processes that represents natural phenomena are usually second order stationary; that is, the first and the second orders moments do not depend on the choice of the time origin(Harvey, 1981). In the periodic process class, two models stand out: PAR (periodic autoregressive) and PARMA (periodic autoregressive-moving average). The PAR (*p*) model adjusts an AR (*p*) model for each period of the series. In a similar way, a PARMA (*p,q*) model consists of an ARMA (*p,q*) model for each period under study. According to (Hipel & McLeod, 1994), in hydrology the PAR (*p*) modeling was developed after the research carried out by (Thomas & Fiering, 1962).

According to (Rasmussen *et. al.*, 1996), the extrapolation of PAR (p) models into PARMA (*p,q*) models is not a trivial task and may not be justifiable, since the autoregressive models perform well. Besides that, according to (Hosking, 1984), the literature shows descriptions of procedures for hydrological series modeling that present long dependence, that is, they have the d parameter of the ARIMA model (differentiation degree), assuming fractionary values. These models are known as ARFIMA (Trevisan*et. al.*, 2000).

The analysis and the modeling of the hydrological series that present a periodic behavior of its probabilistic properties can be performed through periodic autoregressive formulations. These are known as "periodic autoregressive" and PAR (*p*) referenced models, where p is the order of the model. In general, *p* is a vector, $p = [p_1, p_2, ..., p_{12}]$, where each element provides the order of each period (month, in the case of monthly series).

The PAR (*p*) model is mathematically described by:

$$\left(\frac{Z_t-\mu_m}{\sigma_m}\right)=\varphi_1^m\left(\frac{Z_{t-1}-\mu_{m-1}}{\sigma_{m-1}}\right)+\varphi_2^m\left(\frac{Z_{t-2}-\mu_{m-2}}{\sigma_{m-2}}\right)+\ldots+\varphi_{p_m}^m\left(\frac{Z_{t-p_m}-\mu_{m-p_m}}{\sigma_{m-p_m}}\right)+a_t \qquad (1)$$

Z_t	Seasonal series of period *s*
S	Number of periods (*s* = 12 for monthly series)
T	Time index, *t = 1, 2, ..., sN*, year function *T (T = 1, 2, ..., N)* and function of the period *m (m = 1,2, ..., s)*
N	Number of years
μ_m	Seasonal average of the period m
σ_m	Seasonal standard deviation of the period *m*
φ_i^m	*i-th* autoregressive coefficient of the period *m*
p_m	Autoregressive operator order of the period *m*
a_t	Series of independent noises with zero average and variance $\sigma_a^{2(m)}$

The implemented approach uses the moment estimation procedure by the Yule-Walker system of equations to obtain the parameters for each one of the 12 models identified to each month of the year. As stated in (Pagano, 1978) and (Troutman, 1979), for autoregressive structures, the moment estimation procedure is consistent and asymptotic efficient for Gaussian time series, equivalent to the Maximum Likelihood estimation. In this particular approach the 12 models are independently estimated. Therefore, in the case of a PAR(*1*) model for a particular model m, only the autoregressive parameter (φ_1^m) is estimated (besides the noise variance).

In other words, the PAR (*p*) model is adopted in the modeling and simulation of NIE in the BES and thus adjusts an autoregressive model of p order to each period (months) of the historic series of NIE. And this is carried out for each one of the four subsystems that form the Brazilian Interconnected System i.e., Southeast/Mid-West, South, Northeast and North subsystems. For the generation of hydrological scenarios, through the Monte Carlo Simulation a lognormal distribution is adjusted to the residuals((Maceira*et. al.*, 2005); (Spinney & Watkins, 1996)).

4. Final Remarks

The aim of this article was to present the reader with a different view of the Brazilian Electric Sector, *i.e.*, as was noticed in the text, there is an intrinsic relationship between the stochasticity and the activities performed by BES, a major infrastructure sector of the country, which has not been emphasized in every article.

It was evident that the synthetic series of energy/inflows are crucial in determining what is the best way to operate the sector, subsidize decisions about when to expand it or not, thus avoiding cost and/or unnecessary losses. And yet, they are a major factor in determining the short-term price of electricity, since the amount simulated/predicted level of water at the reservoirs in the future will be one of the determinants in setting the short term price.

The importance of this article goes far beyond what was discussed in its essence, it opens a range of possibilities and discussions on how determination being carried out the operation planning, the expansion planning and the electricity spot price. In this sense, the idea of this paper is to provide an intangible result creating a research agenda on the stochastic modeling involving BES. In other words, one should discuss points of improvements to the model adopted by the sector, whether the current model is the most appropriate and discuss points related to the effects of climate change that has occurred over the years on the planet, because, as evidenced changes in rainfall, can cause great damage to BES and consequently damage to the economy.

This way, the idea of this article, much more than presenting the relationship: stochasticity*vs* BES, is to put foward a real problem, where policymakers should be alert not only to the modeling problems that exist today, but also should consider what are the attitudes of the country if any climate event affects in a "unpredicted way" the NIEs of a given subsystem.

Finally, as an extension of this work, is a discussion of the models existing today in BES and used to generate synthetic series for the NEWAVE model and yet, the search for new models and studies that address the occurrence of unexpected climatic events.

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[i] Natural Inflow Energy is the amount of electricity that can be generated by hydroelectric park with water that reaches the hydro. This energy is estimated assuming that the level of the reservoirs is an average level of 65% of its total capacity and a political operation. Remember that this value can change according to the operation policy (Terry, Pereira, Neto, Silva, & Sales, 1986).

[ii] It is important to note that such levels can be of different natures, that is, (i) load level, which is an aggregation of the energy values over a time interval (*e.g.*, months) to separate low demand (load light), medium and high. Broadly, it is the time period in which the characteristics of energy consumption tend to be similar, and (ii) Threshold curve cost deficit, which were regarded as different thermal costs, each with ability of a particular "generation", for example, 5% of the demand for the first stage of thermal deficit (or the "first level"). For more detailssee: (EPE, 2009).