Optimal Taxation of Labor Income and Social Networks

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Abstract

We study optimal tax policy in a model economy where workers find their jobs from their peers in a social network. The unemployment rate is then determined by the dynamic of the labor market, which is governed by the social network. Unemployment results as individuals are unsuccessful in hearing about job opportunities themselves or through their peers in a network. The design of optimal tax policy follows the Ramsey approach. We show that the optimal labor income tax is negatively related to the unemployment rate and it is higher in more connected job network economies.

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1 Introduction

The importance of social networks in labor markets has long been understood. Networking plays a critical role in job searching and in improving the quality of the match between firms and workers. The job network literature indicates that access to information about job opportunities is heavily influenced by social structure and that individuals use connections with others (e.g., relatives, friends, acquaintances) to build and maintain information networks (see Granovetter (1995), and Ioannides and Loury (2004) for a recent survey). Social networks have important implications for the dynamics of employment, as well as, the duration and persistence of unemployment (Calvó-Armengol and Jackson, 2004). As networks might affect economic outcomes, the relevance of social networks for the design of government policies must be recognized and explored.

The literature on optimal labor income taxation, however, has neglected the role of social networks in the labor market and has mainly focused on competitive or job search labor markets. Empirical research indicates that about half of jobs are obtained through networking and the other half are obtained through more formal methods (see Holzer, 1988; Montgomery, 1991; Topa, 2001, Gregg and Wadsworth, 1996; Addison and Portugal, 2001). Well-known results in the theory of optimal labor taxation are that tax rates on labor should be roughly constant, i.e., the optimal labor income tax rates are constant across time and states (Barro, 1979; Kyndland and Prescott, 1980; Chari and Kehoe, 1999), and labor taxes vary positively with employment (Zhu, 1992; Scott, 2007). In this paper we examine if these results survive when the labor market is governed by job networks.

We study optimal tax policy in a model economy where the informational structure of the job networking follows the classic epidemic diffusion model, surveyed recently in Vega-Redondo (2007). We apply the mean field approach, which assumes there are no correlations or neighborhood effects in information transmission, and a network is described by a degree distribution. Our approach amounts to assuming the average state of the network is replicated locally, for every agent, so that the proportion of an agent's peers who are employed is given by the employment rate. The mean-field approach is analytically simple and allows us to calculate well the long-run, average behavior of arbitrary networks, including power-law distributions and networks with the "small-worlds" properties of low diameter and high clustering. As expected, our model predicts that changes in the social network structure will alter the unemployment rate: an increase in the density of social ties generates lower unemployment level.

Information about job opportunities arrives randomly. All jobs are identical and the job arrival process is independent across agents. If the agent is unemployed, she will take the job. On the other hand, if the agent is already employed then he may pass the information along to a friend, relative or acquaintance who is unemployed. Each agent is connected to others through a network. Workers without jobs are in competition for the job information that their peers may pass them. The strength of social ties among workers determines the probability their peers pass job information along.

Unemployment results when individuals are unsuccessful in hearing about job opportunities themselves or through their peers in a network. Agents do not know the employment status of their peers; Job information arrives randomly, and cannot be passed on if it is not needed; in that sense job information may be lost. The unemployment rate is then determined by the dynamic of the labor market, which is governed by the social network. That is, the flow of agents between employment and unemployment status depends on the job arrival and break-up probabilities and a worker's social network contacts. We will consider several different classes of network, and investigate their properties

There are at least three main reasons for studying optimal labor income taxation in this environment. First, one of the most robust and best-studied roles of social networks concerns obtaining employment. There have been a number of studies of how social contacts matter in obtaining information about job openings. Second, labor income tax rates vary substantially over time and across countries and high labor taxes are often seen as one of the causes of high unemployment rates. And, third, our extensions to the specific models used by Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991), with the addition of social networks, enable us to provide a new insight into the relationship between taxes and the labor market dynamics.

This paper embeds the job network model into the general equilibrium framework and the design of optimal tax policy follows the Ramsey approach. This approach to optimal taxation is a solution to the problem of choosing optimal taxes given that only distortionary tax instruments are available. A social planner maximizes its objective function given that agents are in a competitive equilibrium and the optimal path of the planner's fiscal instruments are obtained such that the agent's utility is maximized. We follow the majority of the literature in assuming that there are institutions that effectively solve the time inconsistency problem so that the government can commit to its announced policy.

Our analysis proceeds in three stages. First, we characterize the long-run unemployment rate in the economy, as function of the underlying social network, and job transmission processes. Second, we consider an economy with a representative infinitely lived household. Each household consists of a continuum of family members, which either work or are unemployed. Employed workers receive a wage that is determined competitively, while agents without a job receive an unemployment benefit. Unemployed workers do not search for a job but rather learn about job opportunities through peers in their social network. We derive the optimal labor income tax and show that, under some conditions, the Ramsey optimal policy consists in making the labor income tax decreasing in the unemployment rate. Finally, we explore how different aspects of social networks can affect the design of the optimal tax policy via the determination of the unemployment rate in this economy.

The introduction of labor market frictions through job networks implies that the optimal tax policy should feature some response to unemployment. We show that labor income taxes vary negatively with unemployment and there is a positive relationship between labor income taxes and hours worked. Labor is more inelastically supplied when employment is high and, since the Ramsey planner is required to tax inelastic variables more heavily to minimize tax distortions, labor income tax rates vary positively with hours worked (Zhu, 1992; Scott, 2007).

At the steady state, the number of newly employed agents is exactly equal to the number of newly unemployed agents, and the economy will remain at this level of employment indefinitely. This long run prevalence of employment we take to be the economy's employment rate. This in turn defines the steady state unemployment rate. This steady state unemployment rate is decreasing in the job arrival probability, the job information transmission probability, and is increasing in the job break up probability. Since the optimal labor income tax is decreasing in the unemployment rate, it is positively related to the transmission rate of job information from peers in a particular network. In less connected economies, the unemployment is inefficiently high and the planner faces a tradeoff that calls for responding to unemployment (which reduces households' welfare) by reducing labor income taxation. We show that the optimal income tax is higher in more connected job network economies.

The paper proceeds as follows. In section 2, we present the model economy and characterize a labor market dynamics governed by social networks and exogenous job separation. We discuss how the unemployment rate is affected by job networking where the informational structure of the job networking follows the classic epidemic diffusion model. We derive the optimal labor tax as function of the unemployment rate, which is exogenously given in the agent's problem. Section 3 shows how job networking can affect the optimal labor tax via the unemployment rate. Section 4 concludes.

2 The Model Economy

The model is built on Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991, 1996). Labor is the only factor of production in this economy. Government consumption is exogenously given and it is financed by proportional taxes on labor income. The labor market is characterized by social network and exogenous job separation. The informational structure of the job networking follows the classic epidemic diffusion model, surveyed in Vega-Redondo (2007). The flow of agents between employment and unemployment status depends on an exogenous job separation rate and on a worker's social network contacts. We consider the role of social networks as a manner of obtaining information about job opportunities and consider its implications for the dynamics of employment. Moreover, we determine a law of motion for the workers employed and those without a job, as well an unemployment rate, as a function of job arrival and break-up probabilities and job network structure.

2.1 Network Structures and Job Learning Probability

There is a representative infinitely lived household in the economy. Each household consists of a continuum of family members whose total measure is normalized to one. Time evolves in discrete periods indexed by t. Let the vector s_t describe the employment status of a family member at time t. There are two classes of agents - employed and unemployed workers. If agent i is employed at the end of the period t, then $s_{it} = 1$ and if i is unemployed then $s_{it} = 0$.

The employment rate in the economy at the end of period t is therefore

$$n_t = \int_0^1 s_{it} di$$

Information about job opportunities arrives randomly. Each agent hears about a job opening with probability $\alpha \in [0,1]$. All jobs are identical and the job arrival process is independent across agents. If the agent is unemployed, she will take the job. On the other hand, if the agent is already employed then he may pass the information along to a friend, relative or acquaintance who is unemployed. The rate at which an employed worker passes information to each of her unemployed peers is given by $v \in [0,1]$. In general this is distinct from α, and need not be directly derived from it. The job information process for workers with jobs may be very different that for workers without jobs. Let ρ be the exogenous job break up probability, which is independent across agents.

A job contact network, or social network, is described by a symmetric matrix g, where $g_{ij} \in \{0,1\}$ denotes whether a link exists between agents i and j. That is, $g_{ij} = 1$ indicates that i and j know each other and $g_{ij} = 0$ otherwise. We assume that $g_{ij} = g_{ji}$, meaning that the relationship between i and j is reciprocal.

Because we are considering a continuum of workers, we will focus on large, random networks, that are described by a degree distribution $\{p_k\}_{k=1}^{\infty}$, where p_k is the proportion of agents who have k peers. In general, there may be many networks g consistent with a particular degree distribution $\{p_k\}_{k=1}^{\infty}$. In particular, we study the empty, regular, power-law and geometric degree distributions.

2.2 Job Networking and the Unemployment Rate

We are now in the position to determine the law of motion for the workers employed and the ones seeking for a job. Labor force is normalized to unity. Employment (n_t) is then given by total labor force minus the number of unemployed workers

$$n_t = 1 - u_t \tag{1}$$

The employment rate may be different for agents with different number of links k. The average employment rate can then be expressed as follows:

$$n_t = \int_{k=1}^{\infty} (n_{t,k} p_k)\, dk,$$

where $n_{t,k}$ is the employment rate among agents with k links. Agent who have more links may expect to hear about jobs from their peers more often, and their employment status will evolved differently than that of an unemployed agent.

To analyze the dynamics of employment, we apply the mean field approach, which assumes there are no correlations or neighborhood effects in information transmission. This is obviously an unrealistic assumption. Our approach amounts to assuming the *average* state of the network is replicated *locally*, for every agent, so that the proportion of an agent's peers who are unemployed is given by the unemployment rate (Vega-Redondo 2007). This is untrue, in general. Calvo-Armengol and Jackson (2004) showed that each worker's employment status is correlated with that of his peers, so an agent who remembers his past status could infer the expected employment rates of this peers, and this need not be equal to the average state of the network. The mean field approach relies on an assumption of *homogenous mixing*, that there are no systemic differences between each worker's local neighborhoods. This could be justified by imagining that a worker with k links does not have the same peers period after period, but continually draws new peers, randomly from the network. In that case, because the network is large, he could not infer anything about their employment status beyond the average in the network, and the mean-field approach is correct. Even without that formal assumption, the mean field approach has been shown in simulations to give good answers for the long-run dynamics in the networks we will consider (Vega-Redondo 2007, Jackson 2008).

Following the mean-field approach, and suppressing the subscript t when there is no confusion, the number of employed agents follows the following law of motion:

$$\dot{n_k} = -\rho n_k + (1 - n_k)[(1 - \alpha)k\theta v + \alpha]. \tag{2}$$

The change in the level of employment has three main components. First, ρ percent of agents who are employed will lose their jobs. Second, a fraction α of the unemployed agents will hear of a job themselves. Third, of those unemployed workers who do not hear of a job opportunity themselves, each of their k peers is employed with probability θ, and passes job information at rate v.

The probability their peers are employed (θ) will also depend on the employment status of the network as a whole. According to the mean field approach, we can define this probability in the following way:

$$\theta = \int_{k=1}^{\infty} (n_k \psi_k)\, dk, \tag{3}$$

where ψ_k is the probability an agent's peer has k links, which is given by

$$\psi_k = \int_{k=1}^{\infty} \left(\frac{k p_k}{\int_{k=1}^{\infty} (k\ p_k)\, dk} \right) dk = \int_{k=1}^{\infty} \left(\frac{k p_k}{\langle k \rangle} \right) dk,$$

where $\langle k \rangle = \int_{k=1}^{\infty} (k p_k)\, dk$ is the average degree in the network. Note that $\psi_k \neq p_k$, i.e., the probability your peers have k links is not equal to the proportion of the population that has k links. This is because agents with many links, and a large k, are disproportionately likely to be your peers. Plugging ψ_k into the definition of θ, equation (3), we have

$$\theta = \frac{1}{\langle k \rangle} \int_{k=1}^{\infty} (k\ n_k\ p_k)\, dk.$$

This implies that the probability an agent's peers are employed (θ) depends on the average degree in the network $\langle k \rangle$, the number of links each of these peers have (k), the proportion of agents who have k peers (p_k) and the employment rate among agents with k links (n_k).

In this economy, the unemployment rate u_t follows an exogenous stochastic process and it is a function of the break-up probability (ρ), the job arrival probability (α), the job transmission rate (v) and the degree distribution (p_k). Let $S = \{\rho, \alpha, v, p_k\}$ represent the state of the network and, abusing notation, assume $u_t = u(S)$. Notice that the parameters that describe the job network process and the labor market dynamics enter the household and government problem only through the unemployment rate u_t, as we discuss in the next section.

2.3 Workers, Firms and the Government

In this economy, each member of the household either works during a given time period or is unemployed. There is a measure u_t of unemployed family members and a measure $1 - u_t$ of employed individuals in the household. Employed workers receive a wage that is determined competitively, while agents without a job receive an unemployment benefit. Unemployed workers do not search for a job but rather learn about job opportunities through peers in their social network. In each period of time t, the economy has two goods: a consumption good and labor. Preferences are defined as

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{4}$$

where c and h are consumption and leisure, respectively, β is the discount rate which lies in $(0, 1)$ and $U(.,.)$ is strictly concave with U_c and $U_h > 0$. If an agent has a job, he must choose how much time to allocate to work. Let l_t denote work effort. The time endowment is normalized to 1 so that leisure $h_t = 1 - l_t$.

Employed workers receive a wage w_t for each hour worked and pay a flat labor income tax rate of τ_t. Unemployed households members, u_t, receive an unemployment benefit, b_t. Total real net labor income is given by $(1 - u_t)(1 - \tau_t)w_t l_t$. The sequence of real budget constraint reads as follows:

$$c_t = (1 - u_t)(1 - \tau_t)w_t l_t + u_t b_t + (B_t R_t - B_{t+1}) \tag{5}$$

where the last term refers to bonds purchases and R_t is the real rate of return on bonds. The total household income is divided evenly amongst all individuals, so that family member perfectly insure each other against variation in labor income (Domeij, 2005). Or alternatively we can assume that agents can insure themselves against earning uncertainty and unemployment and, for this reason, wage earnings are interpreted as net of insurance costs (Merz, 1995; Andolfatto, 1996; Faia, 2008).

Production technology is a constant returns specification so that output $y_t = F(l_t)$, where labor is the only factor of production in this economy. Competitive pricing ensures that workers are paid their marginal products, $w_t = F_l(t)$, where $F_l(t)$ denotes the marginal product of labor and w_t is the real wage rate.

We assume that government consumption is exogenously specified and constant, so $G_t = G$.

Taxes on the income from labor finances its consumption and unemployment benefit payments. The government's budget constraint is

$$G + (B_t R_t - B_{t+1}) = \tau_t(1 - u_t)w_t l_t - u_t b_t. \tag{6}$$

In this economy, feasibility requires that the following resource constraint be satisfied:

$$c_t + G = (1 - u_t)w_t l_t. \tag{7}$$

2.4 The Competitive Equilibrium

Let $x_t = (c_t, l_t)$ denote period t allocation, and $x = (x_t)$. We let $\pi = (\tau_t)$ denote government policy, where τ_t represents labor income tax, and $q = (w_t)$ price rule for this economy. Given this description of the economy, we now define competitive equilibrium.

Definition 1 *A competitive equilibrium is a policy π and an allocation x such that given the policy, the resulting allocation maximizes the consumer's utility and satisfies the government's budget constraint, the economy's resource constraint, and market clearing conditions.*

Households choose consumption and labor allocations so as to maximize the present discounted value of utility, equation (4), subject to the their budget constraint, equation (5), and standard nonnegative conditions. We can formulate this problem using a Lagrangian, e.g.:

$$\max_{\{c_t, h_t\}} \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t, h_t) + \lambda_t[(1 - u_t)(1 - \tau_t)w_t(1 - h_t) + u_t b_t + (B_t R_t - B_{t+1}) - c_t] \right\}$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. The first order conditions for consumption and leisure for this problem are respectively:

$$U_c(t) - \lambda_t = 0 \tag{8}$$

$$U_h(t) - (1 - u_t)(1 - \tau_t)w_t \lambda_t = 0 \tag{9}$$

$$-\lambda_t + \beta \lambda_{t+1} R_{t+1} = 0 \tag{10}$$

And the equilibrium conditions can be represented as

$$\begin{aligned}\frac{1}{(1-u_t)(1-\tau_t)}\frac{U_h(t)}{U_c(t)} &= F_l(t) \\ \frac{U_c(t)}{\beta U_c(t+1)} &= R_{t+1}\end{aligned}$$

where the real wage rate is equal to the marginal product of labor, i.e. $w_t = F_l(t)$.

2.5 Ramsey Equilibrium

At the beginning of each period, the government announces its program of tax rate and individuals behave competitively. The objective of the social planner is to choose values of its fiscal instruments such that the agent's utility is maximized. The problem is constrained by the households' and firm's optimization behavior and by the budget of the government. The status of the network, reflected in the economy's unemployment rate, also constrains the planner's problem. The social planner does not directly control the agent's allocations, and the problem is of second-best because the social planner chooses the fiscal instrument that satisfies the optimization restrictions of the private agent, i.e. the first-order conditions of the private agent's problem.

The Ramsey problem is a programming problem of finding optimum within a set of allocations that can be implemented as a competitive equilibrium with distorting taxes. In other words, the Ramsey problem is to choose a process for tax rates $\{\tau\}$, which maximizes social welfare and satisfies (5) and an implementability constraint (see Chari and Kehoe, 1999). We follow the majority of the literature in assuming that the government can commit to follow a long-term program for taxing labor income. We assume that there are institutions that effectively solve the time inconsistency problem so that the government can commit to the tax plan it announces in the initial period.

To derive the implementability constraint, we use conditions (8) and (9) and the intertemporal budget constraint. We multiply the period t budget constraint (5) by its Lagrange multiplier λ_t and sum over t. This yields the following expression:

$$\sum_{t=0}^{\infty} \beta^t \left[U_c(t)(c_t - u_t b_t) - U_h(t)(1-h_t)\right] = 0 \tag{11}$$

A Ramsey equilibrium in this economy is a policy π, an allocation rule $x(\cdot)$ and a price rule q that satisfy the following two conditions: (i) The policy π maximizes (4) subject to the government budget constraint (6) with allocations and prices given by $x(\cdot)$ and q; and (ii) for every π', the allocation $x(\pi')$, the price rule $q(\pi')$ and the policy π' constitute a competitive equilibrium.

Proposition 1 *(Ramsey Allocations) The consumption and labor allocations in a competitive equilibrium satisfy* (7) *and the implementability constraint* (11)*. Furthermore, allocations that satisfy* (7) *and* (11) *can be decentralized as a competitive equilibrium.*

Proof. See Appendix. ■

From our characterization of a competitive equilibrium, we can see that the allocations in a Ramsey equilibrium solve the Ramsey allocation problem of maximizing consumer's utility (4) subject to constraint (7) and (11). The proof of this proposition has two parts: Any competitive equilibrium allocation must satisfy (7) and (11). Conversely, any allocation satisfying (7) and (11) can be decentralized as a competitive equilibrium. Hence, the resource constraint (7) and the implementability constraint (11) completely characterize the competitive equilibrium allocations.

For convenience, write the Ramsey allocation problem in Lagrangian form:

$$\max_{\{c_t, h_t\}} \sum_{t=0}^{\infty} \beta^t W(c_t, h_t, \eta) \tag{12}$$

subject to (7). The function W simply incorporates the implementability constraint into the maximand and is given by

$$W(c_t, h_t, \eta) = U(c_t, h_t) + \eta \left[U_c(t)(c_t - u_t b_t) - U_h(t)(1 - h_t)\right]$$

where η is the Lagrangian multiplier on the implementability constraint, equation (11), and represents the excess burden of taxation, or how tightly the intertemporal budget constraint is binding. The first order conditions for this problem imply that

$$\frac{W_h(t)}{W_c(t)} = F_l(t) \tag{13}$$

The sequences $\{c_t\}_{t=0}^{\infty}$, $\{l_t\}_{t=0}^{\infty}$ and $\{\tau_t\}_{t=0}^{\infty}$ which satisfy (13) constitute a Ramsey equilibrium.

2.6 Optimal Labor Income Taxes

We can use the Ramsey allocation problem to derive some simple results on optimal labor taxes. In order to derive expressions for optimal labor taxes, we make assumptions regarding preferences and consider a set of stylized models where we prove analytically that under some conditions the Ramsey optimal policy consists in making the labor income tax decreasing in the unemployment rate.

Consider the first-order conditions for the Ramsey problem - (12) subject to (7):

$$(1+\eta) + \eta H_c(t) = [\gamma / U_c(t)] \tag{14}$$

$$(1+\eta) + \eta H_h(t) = [\gamma / U_h(t)]\, F_l(t) \tag{15}$$

where η and γ are the Lagrangian multipliers on the implementability constraint and resource constraints, respectively, $H_c(t) \equiv [U_{cc}(t)(c_t - u_t b_t) - U_{hc}(t)(1-h_t)] / U_c(t)$ and $H_h(t) \equiv [U_{ch}(t)(c_t - u_t b_t) - U_{hh}(t)(1-h_t)] / U_h(t)$. Combining (14) and (15), we obtain:

$$\frac{(1+\eta) + \eta H_c(t)}{(1+\eta) + \eta H_h(t)} = \frac{U_h(t)}{U_c(t)} \frac{1}{F_l(t)}$$

Using (8) and (9), we get the following expression for the optimal labor income tax

$$\tau_t = \frac{1}{(1-u_t)} \left[\frac{\eta\, [H_h(t) - H_c(t)] - u_t\, [(1+\eta) + \eta H_h(t)]}{(1+\eta) + \eta H_h(t)} \right] \tag{16}$$

where η is the Lagrangian multiplier on the implementability constraint, equation (11). Notice that (16) is not an explicit expression for the optimal tax rate, since the H_h, H_c depend on endogenous variables.

3 Network Structures and Labor Income Taxes

In this section we study the effects of different network structures on the optimal labor income tax, which in our model economy occurs through the unemployment rate. We study this economy in steady state starting first by analyzing the relationship between job networking and the unemployment rate and, then, the relationship between the unemployment rate and the optimal labor tax.

If the economy converges to a steady state it implies that the change in the level of employment is equal to zero, i.e., $\dot{n}_k = 0$ for all k. The number of newly employed agents of each type k is exactly equal to the number of newly unemployed agents, and the economy will remain at this level of employment indefinitely. This *long run prevalence* (Vega-Redondo 2007) of employment we take to be the economy's employment rate. Setting $\dot{n}_k = 0$ in equation (2), we find that the steady state level of employment (n_k^*) satisfies

$$n_k^* = \frac{\alpha + (1-\alpha)k\theta^* v}{\rho + (\alpha + 1 - \alpha k\theta^* v)} \tag{17}$$

According to equation (3), θ^* is given by

$$\theta^* = \frac{1}{\langle k \rangle} \int_{k=1}^{\infty} (k n_k^*)\, dk. \tag{18}$$

Together, the solution to these two equations for each k gives n_k^*, which can be used to define the economy's long run steady state employment rate

$$n^* = \int_{k=1}^{\infty} (n_k^* p_k)\, dk, \tag{19}$$

and the associated unemployment rate $u^* = 1 - n^*$.

Interestingly, for different degree distributions $\{p_k\}_{k=1}^{\infty}$, the long run steady state employment rate, equation (19), may have different solutions, with different characteristics and implications for optimal labor tax. For instance, consider four networks, as follows:

Example 1 (Empty Network) *As a baseline, we consider the case where $p_0 = 1$, and $p_k = 0$, for all $k > 0$. In this case the only source of job information for each worker is her/himself, and the equilibrium level of unemployment is given by $\alpha / (\alpha + \rho)$.*

Example 2 (Regular Networks) *When $p_k = 1$ for $k = m$ and $p_k = 0$ for all other k, we say the network is* regular*, and every agent has the same number of links m.*

Example 3 (Power Law Networks) *An important class of networks is the* power-law *networks, where $p_k = (\gamma - 1)k^{-\gamma}$. Many models of social networks are described as deriving from linear growth in agents, and preferential attachment in link formation. In these models, we imagine the network growing over time. Workers arrive and choose to form some number of links to the workers already present in the network, with a preference for having links to workers with many links already. This preference is easy to justify, as well connected peers are more likely to be employed themselves, and thus prove to be a valuable source of job information. The limit of this process, as the number of workers goes to ∞, results in a power law degree distribution. A few workers end up with many, many links, while most have relatively few. These networks have a number of attractive features, that match well many empirical observations (Vega-Redondo 2007, Jackson 2008).*

Example 4 (Geometric Networks) *When $p_k = \nu^{1-k} \log \nu$, we say that the network is* geometric. *This corresponds to models of network formation where networks grow, as in power-law networks, but workers form their links without preferential attachment, purely at random among existing workers. This degree distribution has a less heavy tail than the power law distribution, and fewer workers with large numbers of links.*

For each of these possible networks, the behavior of unemployment with respect to the job information process is straightforward. The unemployment rate is decreasing in both the job opportunities arrival probability (α) and the rate at which an employed worker passes information to each of her unemployed peers (v). And, there is a positive relationship between the exogenous job break up probability (ρ) and the equilibrium unemployment rate.

One can also show that the equilibrium unemployment rate is different depending on the network structure and the number of links k an agent has, i.e., agents with more peers will have a lower (individual) unemployment rate. The unemployment rate is always the highest in the empty network, and it falls in regular networks as the number of links (k) each worker has rises. In the case of power law and geometric networks, where there is heterogeneity in the number of links workers have, the equilibrium unemployment rate is decreasing in the number of links k. For these networks, because of the presence of workers with many many links, job information is disseminated more easily, which reduces unemployment.

Finally, the unemployment rate is higher for less connected networks. That is, for any given set of parameters that characterize the state of the network $S = \{\rho, \alpha, v, p_k\}$, we should observe a much higher unemployment rate in a empty network than regular networks, and an even lower unemployment rate in the power law and geometric networks, in particular for greater number of links. Notice that these last two networks can be derived from a growing network process, and are more consistent with empirical social networks, than the regular and empty networks (Vega-Redondo 2007, Jackson 2008). The following proposition summarizes these results.

Proposition 2 *The equilibrium level of unemployment* u (i) *has the following properties:* (a) $\partial u/\partial \alpha < 0$*;* (b) $\partial u/\partial v < 0$ *and* (c) $\partial u/\partial \rho > 0$*, for any network;* (ii) *is constant with respect to the number of links* k *in an empty or a regular (for a fixed* m*) network and it is decreasing in the number of links* k *an agent has in the power law and geometric networks and* (iii) *depends on the network growth process and it is higher in economies with less connected job networks.*

Proof. See Appendix. ■

We illustrate these properties in Figures 1 and 2 for the following set of parameters: $\alpha = 0.5$, $v = 0.05$, $\rho = 0.7$. To parameterize the network structures, we set $m = 2$ in the regular network, so that every worker has two links, $\gamma = 3$ in the power law network, so that the average degree $\langle k \rangle = 2$, and $nu = e$, so that the average degree in the geometric network is also 2. Then, all difference between unemployment in the network comes not from there being, on average, greater or fewer links, but in the different arrangement of links.

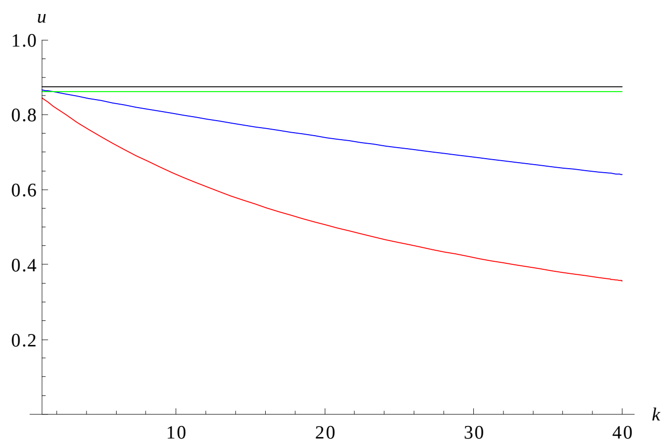

Figure 1 - The unemployment in each network, as the number of links k varies.

Red:Geometric, Blue:Power Law, Green: Regular, Black:Empty.

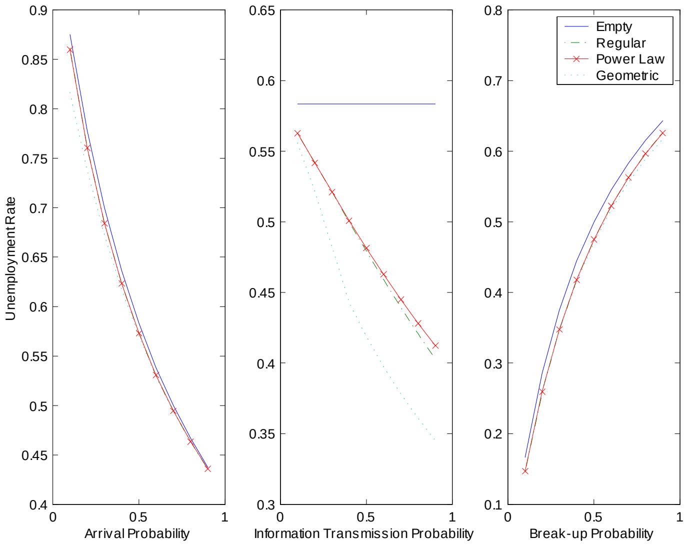

Figure 2 - Equilibrium unemployment in the economy for each network, as α, υ, and ρ vary between 0 and 1.

Next, we investigate the impact of these network dynamics for the optimal labor income tax. In our economy, labor taxation is directly related to the unemployment rate, and so indirectly determined by the structure and properties of the job network. Although the government does not observe how job information is passed from one worker to another, and here we are not arguing that it should, it should recognize the relevance of job networks for the design of the optimal labor income tax, through their impact on the determination of the economy's equilibrium unemployment rate.

The case of additively separable preferences is of particular interest because it allows us to solve for the optimal labor tax analytically. To simplify the presentation consider a quasi-linear utility function

$$U(c_t, h_t) = \ln c_t + \phi \ln h_t \tag{20}$$

When the utility function is additively separable ($U_{ch}(t) = U_{hc}(t) = 0$) in leisure, it implies that leisure is a normal good and labor is taxed.[1] Evaluating (16) for this functional form and assuming that, under the Ramsey plan, the allocations converge to a steady state, we have

$$\tau^* = \frac{1}{(1-u^*)}\left[\frac{\eta(1-h)}{\eta+h} - u^*\right] \tag{21}$$

[1] Basu and Renström (2007) study optimal taxation in an environment with indivisible labor supply, HARA class of preferences with nonseparable leisure.

where u^* is a measure of unemployed family members in a steady state, defined from (19), h is leisure and η is the Lagrangian multiplier on the implementability constraint, equation (11).

In this economy, individuals and family units cannot affect the unemployment rate by their actions. This rate is determined by the dynamics of the labor market, which is governed by social network. But the unemployment rate affects agents' optimal behavior. Agents take into account the proportion of family members unemployed when they make decisions regarding consumption and leisure. Equation (21) suggests that there is a positive relationship between labor income taxes and hours worked $(1-h_t)$. Or conversely, labor taxes vary negatively with leisure (h_t). That is, $\partial\tau^*/\partial h = -\left[\eta(1+\eta)\right] / \left[(1-u^*)(\eta+h)^2\right] < 0$. Also from equation (21), labor taxes vary negatively with unemployment, i.e., $\partial\tau^*/\partial u^* = -\left[h(1+\eta)\right] / \left[(1-u^*)^2(\eta+h)\right] < 0$.

Although in our setup we make a distinction between employment $(1-u_t)$ and hours worked $(1-h_t)$, the intuition for this result follows the same arguments presented by Zhu (1992) and Scott (2007), where the structure of the optimal labor income taxation depends on the elasticity of the labor supply. Labor is more inelastically supplied when employment is high and, since the Ramsey planner is required to tax inelastic variables more heavily to minimize tax distortions, labor income tax rates vary positively with employment and hours worked.

To see this connection more clearly, note that equations (8) and (9) form a system of equations such that c and h can be solved in terms of λ (the Lagrangian multiplier on the household's budget constraint) and ω, where $\omega = (1-\tau)w$. For our purpose we are interested in the compensated labor supply response with respect to a change in ω holding λ constant. We get

$$\frac{\partial(1-h_t)}{\partial\omega_t} = -\frac{(1-u_t)U_{cc}(t)\lambda_t}{U_{hh}(t)U_{cc}(t) - U_{ch}^2(t)} > 0 \tag{22}$$

This expression represents the compensated labor-supply response when the tax rate changes (the substitution effect). This substitution effect captures the distortionary effect of the labor-income tax. That is, a higher labor tax increases leisure and lowers labor supply $(1-h_t)$ and thus lowers the tax base.

For our additively separable utility function, equation (20), the compensate elasticity of labor supply is given by

$$\epsilon_t = \frac{\partial(1-h_t)}{\partial\omega_t}\frac{\omega_t}{(1-h_t)} = (1-u_t)\omega_t(1-h_t)\phi^{-1}\lambda_t \tag{23}$$

There are two effects of the unemployment on the elasticity of labor supply. The unemployment rate impacts this elasticity directly and implies that when unemployment is high, labor is more inelastically supplied. On the other hand, a high unemployment indirectly increases labor supplied, indicating that labor is more elastic. One can show that the net effect of a high unemployment rate on the elasticity of labor supply is negative. That is, labor is more inelastically supplied when unemployment is low (the indirect effect dominates the direct one). Hence, labor income tax rates vary negatively with unemployment.

Given the implications of the network process for the equilibrium unemployment rate, summarized in Proposition (2), we can study how different network characteristics might affect the design of the optimal labor income tax. In economies where the job information process is poor, i.e., low arrival probability, low rate of job information transmission among peers, high job break up probability or low numbers of links, the equilibrium unemployment rate is higher and the Ramsey planner is required to tax labor income at a lower rate. On the other hand, if information about job opportunities is well transmitted among peers, it increases the likelihood of an unemployed worker to hear and get a job. In such an economy, the unemployment rate tend to be lower and the government can implement a higher income tax.

Proposition 3 *When the labor market is governed by social networks, the optimal labor income tax is higher in more connected job network economies.*

Proof. See Appendix. ■

Among the cases we study - empty, regular, power law and geometric - the optimal taxation in the presence of an empty network is a good illustration of one of the extremes faced by the government. In this case, the flow of information about job opportunities among agents is nonexistent. There are no peer effects (no information transmission) and information is lost (if an employed agent hear about another job opening) in this context. Unemployment is higher and, consequentially, it is optimal for the government to tax less those with jobs. To the extent that more and better information is transmitted from an employed worker to his/her unemployed peers, either because agents have more links or because the rate at which such information is transmitted is higher, the required labor income tax is higher. Figure 3 illustrates this result for the geometric network.

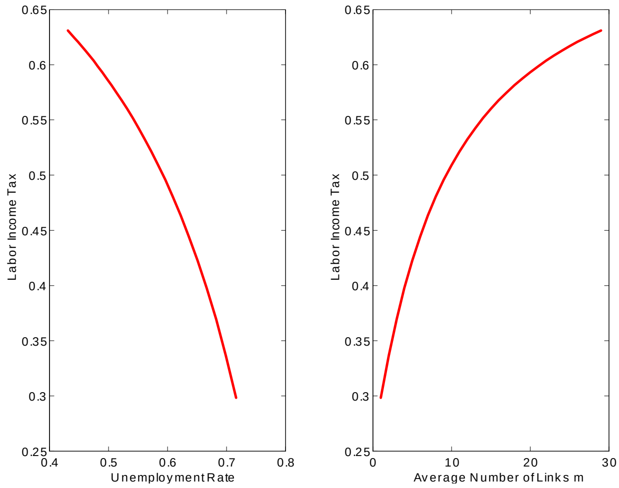

Figure 3 - Labor income tax, unemployment rate and number of links.

(Geometric Network)

4 Conclusion

This paper studies the optimal labor income taxation in the presence of social networks. The unemployment rate is then determined by the dynamic of the labor market, which is governed by the social network. Unemployment results as individuals are unsuccessful in hearing about job opportunities themselves or through their peers in a network. The optimal labor income tax is decreasing in the unemployment rate and the job network process parameters play an important role in determining optimal fiscal policy. The optimal tax is negatively related to the transmission rate of job information from peers in a particular network and it is lower in more connected job network economies. Potential extensions would explore the role of the government spending for the determination of the optimal tax policy, for instance when it varies stochastically, as well allow agents to invest some of their time on building links and connect to peers (endogenous network). We leave this for future research.

Appendix

TO BE ADDED.

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