# "On the marginal social cost of cash-cum-in-kind transfer"

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## Abstract

This paper investigates the marginal social cost of cash-cum-in-kind transfers (MSCKT). Based on a generalization of Wildasin (1984), we characterize the marginal social cost of public funds which have shown to depend on the relation between labor supply and the publicly provided goods (cash-cum-in-kind transfers). To estimate the response of labor supply to these publicly provided goods, and simulate the MSCKT for Brazil, we use the PNAD 2004 database (Brazilian household data). Our simulations suggest that MSCKT increases up to 14% if compared to cases in which cash-cum-in-kind transfers have their effects ignored on labor supply response on the part of individuals.

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## 1. Introduction

For decades, economists have been posing questions on optimum levels of public spending. In view of its great relevance for redistributive policies purposes, the issue of social costs associated to public expenditures requires further analysis. This paper aims at contributing to the specific characterization of the marginal social cost of cash-cum-in-kind transfers by estimating its magnitude using Brazilian data. We first set a scene for this paper with a brief chronological overview of the literature.

A classic definition of the optimum level of public expenditure is provided by Samuelson (1954): $\sum MRS = MRT$ , which is to say that the sum of the marginal benefit of a public good ( $\sum MRS$ ) equals the marginal ratio of substitution of this good in exchange for a private good of reference (MRT), which is usually cash. In other words, the marginal social benefit of a public good is a straightforward consequence of the private good quantity people are willing to give up in exchange for a specific public good, whereas marginal cost is the marginal transformation ratio between public and private goods (MRT). When these two sides of the equation are equalled ( $\sum MRS = MRT$ ), the optimum level of expenditure is obtained.

Samuelson's approach implicitly assumes that governments raise the revenue necessary for financing public projects through lump-sums. Provided that lump-sums are unusual in practice, the distortive effect of increased taxation on taxpayers preferences should be taken into consideration. To capture such effect, Pigou (1947) proposes a term of costs incurred by consumers *above and beyond* the monetary cost of tax measured by the private good of reference. Pigou´s argument can be illustrated by including a multiplicative term (MCF, Marginal Cost of Public Funds - denoted as indirect burden on society)[1] in addition to the direct marginal costs within the determination of the optimum quantity of a public good:

$$\sum MRS = MRT \times MCF \tag{1}$$

[1] Browning (1976) further develops the concept introduced by Pigou defining it as the *marginal cost of public fund* (MCF).

Note that when a government is fully financed by lump-sum taxes, MCF is equal to one. Ballard and Fullerton (1992) identify two distinct methodologies for measuring the marginal cost of funds: the 'Pigou-Harberger-Browning' and 'Stiglitz-Dasgupta-Atkinson-Stern' approaches. Within the 'Pigou-Harberger-Browning' approach, MCF has to be greater than one given the indirect costs of distortive taxes which increase costs associated to the provision of public goods. On the other side, under the alternative 'Stiglitz-Dasgupta-Atkinson-Stern' approach, MCF can be lesser than one in some situations, which would imply lesser costs if compared to the first-best situation.

Noteworthy MCF estimations, based on the first approach mentioned above, were done by Browning (1976, 1987) using data from the U.S. federal tax system. In short, the author characterizes optimal levels of public goods provision which is dependent of the marginal tax rate, rate progress, and the labor supply's compensated elasticity. However, Diamond and McFadden (1974), have challenged the merits of different measures of dead-weight loss and the indirect marginal cost by unit of revenue used in those papers. On the other side, Fullerton (1991) argues that the incongruence between Browning's estimations and other authors were actually differences in the definition of MCF and not differences among parameters as alleged before. Anyhow, within those approaches MCF is necessary greater than one.

Accordingly, without naming it as such, two works have managed to isolate the marginal cost of public fund and benchmarked a new line of thought. Stiglitz and Dasgupta (1971) isolated and observed that the term is lesser or greater than one if the labor supply curve is backward bending. Atkinson and Stern (1974) also isolated MCF but further decomposed it into two parts: *distortive effects* and *revenue effects*. Distortive effects depend on the effects of substitution which make the public project always less attractive. Revenue effects on the other side, account for effects derived from the changed income due taxation and might reinforce the substitution effect like, for example, advocating against the public project implementation (very common event when income tax is considered). Given that revenue effects of taxation may increase labor supply, and consequently government revenue, they may also decrease the marginal costs of public fund. Stuart (1984), Ballard, Shoven and Walley (1985), and Ballard and Fullerton (1992) also apply this approach when estimating MCF.[2]

Wildasin (1984) contributes to the literature by observing that the welfare evaluation of public expenditure should take into consideration the effect of incremental public provision on the demand of taxed goods. Moreover, the author shows that this effect is not insignificant regardless of whether demand is ordinary or compensated. However, only one publicly provided good is considered in this model. In empirical terms, Conway (1997) tries to estimate the effect of public goods provision on the labor supply of individuals' response as suggested by Wildasin (1984). The author finds public expenditures to be significant on the determination of taxed good demand (labor supply response). This result suggests that MCF calculation should incorporate the effect of incremental public provision on the demand of taxed goods, otherwise MCF is underestimated. We do generalize his model in this paper to estimate the marginal social cost of a bundle (cash-cum-in-kind transfers) of publicly provided goods (MSCKT).

This is an interesting generalization justified by the great array of Brazilian social programs, such as the *Bolsa Escola*, that can be considered a bundle of two goods: cash transfer (financial aid) and the publicly provided good (public school).[3] We propose to estimate the marginal social cost of publicly provided goods when taxpayers are distortedly taxed but can receive subsidies if consuming the public good. As such, the objective of this paper is twofold: i) to characterize the solution of marginal social cost of cash-cum-in-kind transfer, and ii) to empirically show how different types of goods provided by the public sector affect the response of agents – these are the basis for estimating the marginal social cost within the Brazilian public sector (MSCKT).

We consider individual transfers (*Bolsa Familia*, *PETI*, public pension, etc.) as public expenditure retained by individuals and not as an involuntary income like in Conway (1997). In doing this, we intend to separate the effects

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[2] Slemrod and Yitzhaki (2001) argue that the marginal cost of public fund should be complemented by a symmetry concept named marginal benefit of public good (MBP) that indicates the utility for individuals in amount of dollars spent. We discuss the works related to our main question. See also Gahvari (2007) for cases of heterogeneous agents.

[3] Gahvari and Mattos (2007) show that this program can be redistributive without distortions.

on labor supply associated to expenditures retained by individuals from other aggregated government expenditures like healthcare, safety, roads, etc. This is only possible because Brazilian data differ from the equivalent U.S. data in terms of transfers' complementarities. Consider the *Bolsa Familia* Program for instance. This program requires children to be enrolled and regularly attend public schools in order to receive cash transfers. Another type of complementary transfer is a program called PETI which requires students to attend extracurricular activities after the period of regular classes. It is then reasonable to expect that individuals do respond differently when exposed to a combination of public spending such as these two descriptive examples. This scenario allows precise analysis of the cost-benefit relation within transfer programs which expand theoretical and empirical frameworks used in public redistributive policies.

This works is organized as follows. The next section characterizes the solution to the problem of homogenous and heterogeneous agents. Section 3 presents the estimations and section 4 the simulation of the marginal social cost of publicly provided goods using Brazilian data. Section 5 concludes the discussion and addresses the implications of results for public policies.

**2. The Model**

Wildasin's (1984) model, mentioned in the previous section, can be extended to include a bundle of goods provided by the government with different benefits and different marginal costs. These goods can be either physical or monetary, which allows for a deeper analysis of the effect on welfare created by different social public programs. First, suppose an economy with H identical consumers and a twice differentiated and strictly quasi-concave utility $u(x_0, x_1, ..., x_n, G_1, G_2)$ defined on the consumption of private goods ($x_i \geq 0$), supply factors ($x_i \leq 0$), and consumption of public Goods $G_1$ and $G_2$ with their specific benefits and marginal costs. Being $G = G_1 + G_2$ the total quantity of goods provided by the government and a the proportion of Good 1 in this bundle in a way that $G_1 = aG$ e $G_2 = (1- a)G$.

Only good 1 is taxed as $q_1 = p_1 + t_1$, being $t_1$ the tax rate, and Good 0 (zero) the numeraire with $p_0 = 1$. For the remaining goods, consumers' prices are equal to producers' prices $p_i$. Each individual faces the problem of selecting a vector consumption of private goods $x = (x_0,...,x_n)$ to maximize her utility subjected to the following budget constrain, $t_1 x_1 + \sum_{i=0}^{n} p_i x_i = Y$ , where Y is exogenous income (virtual income) in numeraire unities. The private production is competitive and subjected to a linear technology; in such a way that equilibrium price vector p is constant.

Demand functions $x_i(q_1, p_2,..., p_n, Y, G_1, G_2)$ and the indirect utility function are outcomes of the solution for the individuals' maximization problem. As in Wildasin (1984), let $MRS_j$ be the marginal substitution rate between public Good $G_j$ and the numeraire, $MRS_j = \partial u/\partial G_j / \partial u/\partial x_0 = \partial v/\partial G_j / \partial v/\partial x_0$ for j =1, 2 and, consequently, the derivative of the ordinary demand function for good i is $\partial x_i/\partial G_j = \partial x_i/\partial G_j\big|_{\bar{u}} + MRS_j\, \partial x_i/\partial Y$ .

Let $X_1 = Hx_1$ be the aggregated demand of Good 1. To balance its budget, the government chooses $t_1$ such that $t_1 X_1 = c_1(G_1) + c_2(G_2)$.

Given that, $dq_1/dG = dt_1/dG$ , $dc_1/dG_1 = MRT_1$ , $dc_2/dG_2 = MRT_2$ , $dG_1/dG = \alpha$ and $dG_2/dG = 1-\alpha$ , we can obtain the MSCKT,

$$\frac{dt_1}{dG} = \frac{\alpha[MRT_1 - t_1 \,{}^{\partial X_1}\!/_{\partial G_1}] + (1-\alpha)[MRT_2 - t_1 \,{}^{\partial X_1}\!/_{\partial G_2}]}{X_1 + t_1 \,{}^{\partial X_1}\!/_{\partial q_1}} \qquad (1)$$

We use the aggregated utility $H_v$ as an indicator of welfare, in the same way as Wildasin (1984). An increment in the quantity of the provided public good is desirable if $H(dv/dG) > 0$ , or equivalently if $(H/v_Y)(dv/dG) > 0$ , being $v_I = \partial v/\partial I > 0$ the marginal utility of income. Using Roy's identity ($(\partial v/\partial q_1)/v_I = -x_1$), we have the following optimal welfare criteria for the provision of a bundle of goods:

$$\frac{H}{v_Y}\frac{dv}{dG} = \alpha \sum MRS_1 + (1-\alpha)\sum MRS_2 - \frac{\alpha\left[MRT_1 - t_1 \,{}^{\partial X_1}\!/_{\partial G_1}\right] + (1-\alpha)\left[MRT_2 - t_1 \,{}^{\partial X_1}\!/_{\partial G_2}\right]}{1 + (t_1/q_1)\varepsilon_1} \qquad (2)$$

where $\varepsilon_1 = \partial \log x_1 / \partial \log q_1 = (q_1/x_1)(\partial X_1/\partial q_1)$ is the ordinary elasticity of Good 1 demand in relation to its own price. At the optimum level G (second-best), expression (3) is zero. In this expression, the first right side term is the marginal benefit for society given public Good 1, the second term is the marginal benefit given public Good 2, and the third and forth terms are the marginal costs associated to the provision of public Goods 1 and 2, respectively. The model can be easily extended for *n* public Goods and the resulting expression would similarly be a difference between the marginal benefits sum of each good and the sum of marginal costs of provision.[4]

There are two terms In expression (2) that make the marginal change in welfare simply different from $\sum MRS_1 + \sum MRS_2 - MRT_1 - MRT_2$. If raised G increases or decreases $X_1$, government revenue will also increase or decrease so $t_1(\partial X_1/\partial G_j) \neq 0$ for *j*=1, 2. Additionally, the denominator presents $1 + \frac{t_1}{q_1}\varepsilon_1$ . The term $(t_1/q_1)\varepsilon_1$ is negative when Good 1 is a normal good or when it is a factor with an upward sloping supply curve. Assuming that the first term is zero $(t_1(\partial X_1/\partial G_j = 0)$, j=1,2), $MRT_1$ + $MRT_2$ still underestimates the true social marginal cost of the public Good. On the other side, if Good 1 is a taxed factor with a backward bending curve, $(t_1/q_1)\varepsilon_1$ will be greater than 1 and $MRT_1$ + $MRT_2$ overestimates social marginal costs. In this case, we can have $MCF < 1$, in accordance with Atkinson and Stern (1972).

### 3. Empirical Implementation

The methodology used to estimate the supply job response to variations in the public provision of monetary and non-monetary goods while taking into consideration net income and other income sources (e.g. endogenous

[4] Deriving this expression in relation to α, we obtain $d((H/v_I)dv/dG)/d\alpha = (\sum MRS_1 - \sum MRS_2) - (MRT_1 - t_1\,\partial X_1/\partial G_1) - (MRT_2 - t_1\,\partial X_1/\partial G_2)/(1 + t_1\varepsilon_1/q_1)$, that is, if people assign a greater value to public Good 1, the marginal benefit increases according to increases in a, given that a is the proportion of Good1 in the bundle of goods provided by the government. However, if people assign a greater value to public Good 2, increases in a decreases the marginal benefit given that the proportion of Good 2 decreases. On the other side, if the marginal cost of public Good 1 $((MRT_1 - t_1\,\partial X_1/\partial G_1)/(1 + t_1\varepsilon_1/q_1))$ is greater than public Good 2, increases in a also increase the marginal cost in equation (9) given that it increases the proportion of G1 in the bundle – on the contrary, an increase in a decreases the marginal cost.

income) is proposed by MaCurdy et al (1990) and implemented by Conway (1997). We conveniently choose a linear function of labor supply[5]:

$$h_i = \beta w_i + \delta Y_i + \varphi T_i + \phi G + X\Gamma + \varepsilon \qquad (3)$$

where $h_i$ corresponds to the labor supply of individual *i* measured by monthly hours, $X_i$ is the explicative (control) variables vector, $w_i$ is the individual's income (declared monthly income)[6], $Y_i$ (virtual income) corresponds to other sources of income (rents, investments, inheritance, other family members income, etc.), *G* denotes the public expenditure variable (state public expenditure), $T_i$ denotes individual transfers done by the public sector and appropriated by individual i (*Bolsa-Familia*, Social Security, etc.) ), β, γ, φ, δ and Γ are coefficients to be estimated, and ε is a random term assumed to be independent and identically distributed $N(0,\sigma^2)$. For our analysis, the variables used as control for individuals ($X_i$) are: age, squared age, schooling, squared schooling, number of school age children (6 to 15 years), number of children < 6 years old (in some cases as described below), race dummies, marital status dummies (married or not- in cases of non-stratified samples), job formality dummy, public sector job dummy, length of time living in the State (> two years) and local of residence (urban or rural area), and the change in the State GDP per capita. These variables are based on the related literature (Conway, 1997 and Avelino and Menezes-Filho, 2003). The sample is restricted to people between 25 and 60 years old excluding self-employed and employer.

However, $E(w_i\varepsilon_i), E(Y_i\varepsilon_i), E(T_i\varepsilon_i)$ is easily $\neq 0$ when it is not possible to determine the causality between tax variations, net wage and/or other income sources and the corresponding response of the worked hour variable. That could generate biased estimators of β, γ, φ, δ and ρ (Greene, 2005).

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[5] The income tax structure in Brazil is progressive which makes the agents restriction "linear by parts." Two options for estimating the model include: the maximization by likelihood of the complete model (Burtless and Hausman, 1990, and Moffit, 1990) and a two-stage procedure that considers the families' income endogenous, which results in the "linearization" of the budget (Killingsworth, 1983 and Rosen 1976a, b). We use the second procedure, more simple and more used in the literature, but above all, Monte Carlo's experiments show that the two procedures produce similar estimations (Triest, 1987).

[6] Ideally, we should obtain the individual's net income; however, given that most of the sample (more than 80%) is exempt from income taxes and the incidence of other taxes depend of non-observable factors, we opted not to assume that the monthly declared income a reasonable proxy for net income.

To address this problem, we identify at least four instruments, correlated with the possible endogenous variables: a) the decision to work, b) the virtual income (Y), c) the individual's wage (w), and d) transfers (T) received by the individuals– but not directly correlated with the numbers of hours worked. In this paper, we use the following instruments to serve such purpose: for the decision to work we use the State unemployment rates (tx_des) and the number of children less 6 years old as instruments. The first variable captures the regional effect of the activity level that influences the individual's probability of getting a job. The second was used only within the subsample *single-women* and follows Heckman (1979) where women with young children choose not to work. And finally, for the first stage regression (self-selected) which the dependent variables are the virtual income (Y), the wage (w), and the transferred income (T), we use the instruments suggested by Conway (1987) and Mroz (1987): cubic polynomials of age and schooling, and country-regions dummies (North, NorthEast, South-East, MidWest and South). In the second stage we run a self-selected model specifying a bootstrap with 50 repetitions for residuals covariance matrix estimation.

### *3.a. Database*

The analysis is drawn from monthly data of the 2004 National Household Sample Survey (PNAD), carried out by the Brazilian Institute of Geography and Statistics (IBGE). We also use 2004 Public Finance and Economic data at State level from the Institute of Research and Applied Economics (IPEA) obtained through their website: state unemployment rate, state GDP per capita growth (2005-2004), total state public expenditure, state public expenditure on education and culture, on health, on housing, and remainder. The last variable is calculated through the difference between state expenditure (total) and the sum of the following public expenditures mentioned above: education, health and housing. Table 1 defines each variable and presents the descriptive statistics. State expenditures are divided by population.[7]

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[7] Population is powered by (-0.8) and multiplied it by expenditure – this expenditure structure better fits our data. In fact, we could used any ponderation (population exponent between 0 and -1) from total expenditures (G*pop^0) up to using expenditure per capita (G*(pop^-1)), depending on how public the good being provided by the public sector is. If it is a true public good, the first option ought to be adopted (G), if it is a good appropriated by individuals, expenditure per capita is the best option.

A quick scan of Table 1 shows that unconditionally comparing men and women, men do work more (172 hours and 150 hours per month respectively), in average receive higher wages (R$5.6 and R$4.6) and receive less transfers (R$153 and R$188) and less virtual income (R$474 and R$815). There is no much age difference between women and men (38.3 and 35.5) and men in average have less years of schooling (7.2 and 8). By looking at Table 1 we also note that married men work more (173 hours/month) receive less transfers (T=88), have smaller virtual income (Y=R$419) and larger income (w=R$5.8) than single men (168 hours/month, T=360, and R$650, and R$5.2 respectively). On the other side, single women do work more (154 hours/month) and in average receive less (w=R$ 4.5) than married women (148 hours/month and R$ 4.6 respectively). Also, single women receive more transfers than married women (T=R$262 and R$165 respectively) but have smaller virtual income (Y=R$458 and R$1,065 respectively).

**Table 1 - Variables Definition and Descriptive Statistics Average/DP**

| Definition | Men | Married Men | Single Men | Women | Married Women | Single Women |
|---|---|---|---|---|---|---|
| Monthly Hours | 172.419 | 173.524 | 168.935 | 150.887 | 148.361 | 154.491 |
| | 28.984 | 28.109 | 31.335 | 43.119 | 44.210 | 41.247 |
| T | 153.652 | 88.189 | 360.113 | 182.298 | 125.814 | 262.884 |
| | 585.448 | 355.178 | 984.786 | 616.523 | 504.268 | 740.667 |
| w | 5.695 | 5.829 | 5.273 | 4.625 | 4.657 | 4.580 |
| | 22.264 | 22.264 | 22.264 | 7.995 | 8.083 | 7.867 |
| Y | 474.724 | 419.082 | 650.209 | 815.089 | 1,064.779 | 458.854 |
| | 1,068.542 | 791.821 | 1,650.778 | 1,477.348 | 1,563.835 | 1,261.584 |
| G | 3,176.137 | 3,174.101 | 3,182.559 | 3,215.536 | 3,233.070 | 3,190.519 |
| | 1,149.142 | 1,148.319 | 1,151.777 | 1,144.572 | 1,135.852 | 1,156.483 |
| Health Expenditure | 377.747 | 377.350 | 379.000 | 381.282 | 379.225 | 384.218 |
| | 147.006 | 146.939 | 147.217 | 149.429 | 148.152 | 151.188 |
| Housing Expenditure | 45.112 | 44.874 | 45.863 | 46.987 | 45.648 | 48.897 |
| | 108.448 | 108.104 | 109.527 | 113.106 | 110.109 | 117.228 |
| Education Expenditure | 494.891 | 494.460 | 496.250 | 499.480 | 502.933 | 494.553 |
| | 254.651 | 254.531 | 255.037 | 252.897 | 254.269 | 250.855 |
| Expenditure - Remainder | 2,309.906 | 2,308.582 | 2,314.082 | 2,338.166 | 2,355.597 | 2,313.299 |
| | 843.848 | 843.525 | 844.899 | 838.213 | 833.473 | 844.339 |
| Schooling | 7.255 | 7.059 | 7.874 | 8.019 | 7.824 | 8.297 |
| | 4.781 | 4.732 | 4.880 | 4.832 | 4.805 | 4.857 |
| Squared Schooling | 74.203 | 71.176 | 83.750 | 85.681 | 82.469 | 90.263 |
| | 75.976 | 74.188 | 80.618 | 80.431 | 79.425 | 81.630 |
| Age | 38.358 | 39.242 | 35.570 | 38.572 | 38.499 | 38.677 |
| | 8.768 | 8.644 | 8.571 | 8.505 | 8.126 | 9.018 |
| Squared Age | 1,548.201 | 1,614.630 | 1,338.698 | 1,560.173 | 1,548.234 | 1,577.206 |
| | 711.931 | 74.188 | 677.269 | 689.569 | 657.287 | 732.858 |
| Children < 6 Years Old | 0.451 | 0.561 | 0.102 | 0.322 | 0.383 | 0.234 |
| | 0.738 | 0.785 | 0.397 | 0.609 | 0.650 | 0.534 |
| Children between 7 and 14 Years Old | 0.740 | 0.892 | 0.261 | 0.755 | 0.879 | 0.578 |
| | 0.978 | 1.015 | 0.648 | 0.973 | 1.004 | 0.898 |
| Unemployment Rate | 0.151 | 0.151 | 0.152 | 0.151 | 0.150 | 0.152 |
| | 0.027 | 0.027 | 0.026 | 0.027 | 0.027 | 0.026 |
| State per capita growth (2005-2004) | 0.146 | 0.146 | 0.146 | 0.145 | 0.144 | 0.147 |
| | 0.040 | 0.040 | 0.040 | 0.040 | 0.040 | 0.041 |
| N | 48,207 | 34,254 | 13,953 | 72,196 | 48,611 | 23,585 |
| N with Hours > Zero | 36,699 | 27,864 | 8,835 | 32,018 | 18,824 | 13,194 |
| **Dummies** | | | | | | |
| Married | 0.759 | 1.000 | 0.000 | 0.588 | 1.000 | 0.000 |
| | 0.428 | 0.000 | 0.000 | 0.492 | 0.000 | 0.000 |
| North Region | 0.101 | 0.102 | 0.100 | 0.097 | 0.093 | 0.103 |
| | 0.302 | 0.302 | 0.300 | 0.296 | 0.291 | 0.304 |
| Northeastern Region | 0.269 | 0.269 | 0.272 | 0.251 | 0.238 | 0.271 |
| | 0.444 | 0.443 | 0.445 | 0.434 | 0.426 | 0.445 |
| Southeastern Region | 0.341 | 0.338 | 0.351 | 0.344 | 0.344 | 0.345 |
| | 0.474 | 0.473 | 0.477 | 0.475 | 0.475 | 0.476 |
| Southern Region | 0.171 | 0.175 | 0.159 | 0.186 | 0.205 | 0.157 |
| | 0.376 | 0.380 | 0.366 | 0.389 | 0.404 | 0.364 |
| White | 0.482 | 0.486 | 0.468 | 0.531 | 0.553 | 0.500 |
| | 0.500 | 0.500 | 0.499 | 0.499 | 0.497 | 0.500 |
| Formal Job | 0.593 | 0.612 | 0.533 | 0.386 | 0.372 | 0.406 |
| | 0.491 | 0.491 | 0.499 | 0.487 | 0.483 | 0.491 |
| Public sector | 0.106 | 0.110 | 0.093 | 0.187 | 0.197 | 0.174 |
| | 0.308 | 0.313 | 0.291 | 0.390 | 0.397 | 0.379 |
| Instate residence ≥ 2 Years Old | 0.314 | 0.327 | 0.270 | 0.294 | 0.302 | 0.283 |
| | 0.464 | 0.469 | 0.444 | 0.456 | 0.459 | 0.451 |
| Urban region | 0.884 | 0.883 | 0.888 | 0.934 | 0.916 | 0.959 |
| | 0.320 | 0.322 | 0.316 | 0.248 | 0.277 | 0.198 |

### *3.b. Results*

Tables 2A, 2B, and 3 present the summary of the results of specification (3) for men and women separately (complete estimations are available upon request). Table 2A presents the results for all *women* and *women* *without children* corrected by the self-selection decision to work. Table 2B separates women according to marital status: single versus married.[8] The summary of estimations concerning job entry decision is shown at the lower portion of Table 2B. Wage is insignificant for the sample containing all *women* and for *single-women* when total public expenditures (G) and Transfers (T) are included. For *women without children*, wage positively influences labor supply (estimated elasticity equals to 0.1, column 7) and negatively for married (elasticity -0.21, column 7). Income derived from transfers (T) is significant and negative for all estimations (see for instance, elasticity equals to -0.06 in column 2). Virtual income (Y) is positive in most of the specifications and, although disconcerting, this value is commonly found in estimations that consider progressive income taxation and do not implicitly impose $\partial h/\partial Y < 0$ (MaCurdy et al, 1990). It is worth noting that this value for women is negative in Conway's (1997) estimations, however, the author considers transfers as part of virtual income – those are estimated separately here. For *single women*, when the variable transfer (T) is omitted, the effect of virtual income changes to negative (column 17).

[8] As mentioned before, the instruments used are cubic age, cubic schooling, and region dummies. In this case, the first stage runs w, Y, and T corrected by the decision to work within the control instruments. The instrument used for the agent's decision to work is the in-State unemployment rate (except for single women which the number of children was also used). In the second stage, we compute the value of these variables in the regression where worked monthly hours is the depend variable which corrects the standard deviation.

**Table 2 A - Summary of 2-stage regression for Women (corrected selection bias)**

(w, T, Y and the decision to work as endogenous)

| Dep: Hours/Month | Women | | | | | Women - no children | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **no G, T** | **no G** | **no T** | **Total State Public Expenditure** | **Public State Expenditure - Parts** | **no G, T** | **no G** | **no T** | **Total State Public Expenditure** | **Public State Expenditure - Parts** |
| | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** | **(6)** | **(7)** | **(8)** | **(9)** | **(10)** |
| | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** |
| w | -7.290*** | -2.948** | -3.797*** | 0.879 | -1.092 | -0.529 | 2.730** | 0.326 | 3.333** | 1.255 |
| | (0.881) | (1.334) | (0.673) | (1.136) | (1.484) | (0.852) | (1.325) | (0.699) | (1.294) | (1.629) |
| T | | -0.041*** | | -0.051*** | -0.041*** | | -0.034*** | | -0.032*** | -0.024** |
| | | (0.010) | | (0.012) | (0.012) | | (0.011) | | (0.010) | (0.011) |
| Y | 0.044*** | 0.025*** | 0.022*** | 0.002 | 0.012** | 0.006 | -0.005 | -0.003 | -0.012** | -0.006 |
| | (0.005) | (0.007) | (0.004) | (0.006) | (0.006) | (0.004) | (0.005) | (0.004) | (0.005) | (0.006) |
| G | | | 0.003*** | 0.002*** | | | | 0.002*** | 0.002*** | |
| | | | (0.000) | (0.000) | | | | (0.000) | (0.000) | |
| G - Health | | | | | 0.035*** | | | | | 0.029*** |
| | | | | | (0.005) | | | | | (0.006) |
| G - Housing | | | | | -0.012*** | | | | | -0.006 |
| | | | | | (0.004) | | | | | (0.007) |
| G - Education | | | | | -0.001*** | | | | | -0.001** |
| | | | | | (0.000) | | | | | (0.000) |
| G - Remainder | | | | | 0.002*** | | | | | 0.001 |
| | | | | | (0.001) | | | | | (0.001) |
| **Probit - reduced form** | | | | | | | | | | |
| G | | | -0.000 | -0.000* | | | | -0.000 | -0.000 | |
| | | | (0.000) | (0.000) | | | | (0.000) | (0.000) | |
| G - Health | | | | | -0.000** | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| G - Housing | | | | | -0.000 | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| G - Education | | | | | 0.000*** | | | | | 0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| G - Remainder | | | | | -0.000 | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| N (2 stage) | 72,542 | 72,542 | 72,542 | 72,542 | 72,542 | 28,748 | 28,748 | 28,748 | 28,748 | 28,748 |

note: 0.01 - ***; 0.05 - **; 0.1 - *;
standard errors in parenthesis

**Table 2 B - Summary of 2-stage regression for Women (corrected selection bias)**
(w, T, Y and the decision to work as endogenous)

| Dep: Hours/Month | Married Women | | | | | Single Women | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **no G, T** | **no G** | **no T** | **Total State Public Expenditure** | **Public State Expenditure - Parts** | **no G, T** | **no G** | **no T** | **Total State Public Expenditure** | **Public State Expenditure - Parts** |
| | **(11)** | **(12)** | **(13)** | **(14)** | **(15)** | **(16)** | **(17)** | **(18)** | **(19)** | **(20)** |
| | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** |
| w | -4.123** | 11.514*** | -6.547*** | -7.120*** | -15.659*** | -0.628 | 0.966 | 0.996 | 1.060 | -0.955 |
| | (1.893) | (0.829) | (0.675) | (1.168) | (2.208) | (1.046) | (1.420) | (1.644) | (1.721) | (1.250) |
| T | | -0.152*** | | 0.010 | 0.074*** | | -0.015** | | -0.008 | -0.008 |
| | | (0.014) | | (0.017) | (0.026) | | (0.008) | | (0.012) | (0.009) |
| Y | 0.013 | -0.040*** | 0.029*** | 0.032*** | 0.059*** | 0.014* | 0.005 | -0.011 | -0.012 | 0.003 |
| | (0.009) | (0.005) | (0.004) | (0.006) | (0.008) | (0.008) | (0.011) | (0.011) | (0.013) | (0.009) |
| G | | | 0.004*** | 0.004*** | | | | 0.002*** | 0.002*** | |
| | | | (0.000) | (0.000) | | | | (0.001) | (0.001) | |
| G - Health | | | | | 0.079*** | | | | | 0.019*** |
| | | | | | (0.007) | | | | | (0.006) |
| G - Housing | | | | | -0.003 | | | | | -0.004 |
| | | | | | (0.005) | | | | | (0.006) |
| G - Education | | | | | -0.002*** | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| G - Remainder | | | | | 0.002** | | | | | 0.000 |
| | | | | | (0.001) | | | | | (0.001) |
| **Probit - reduced form** | | | | | | | | | | |
| G | | | | -0.000* | | | | -0.000 | -0.000 | |
| | | | | (0.000) | | | | (0.000) | (0.000) | |
| G - Health | | | | | -0.000*** | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| G - Housing | | | | | -0.000 | | | | | -0.000* |
| | | | | | (0.000) | | | | | (0.000) |
| G - Education | | | | | 0.000* | | | | | 0.000*** |
| | | | | | (0.000) | | | | | (0.000) |
| G - Remainder | | | | | 0.000 | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) |
| N (2 stage) | 48,611 | 48,611 | | 48,611 | 48,611 | 23,931 | 23,931 | | 23,931 | 23,931 |

note: 0.01 - ***; 0.05 - **; 0.1 - *;
standard errors in parenthesis

Results regarding public non-appropriated expenditures (G) seem to corroborate with the hypothesis that aggregated expenditures positively affect labor supply. That is, public Good (G) is complementary to the number of hours worked in the case of women. Moreover, healthcare disaggregated expenditures negatively, but education positively, affects women's decision to work (in all sub-samples). Once in the job market, all women are more positively affected by healthcare and negatively by education expenditures. That portrays a consistent history: increased expenditure in education increases women's chance to get a job, however, once in the job market, those expenditures reduce labor supply in terms of hours worked /month probably because of their increased productivity. On the other hand, still in the case of women, increased healthcare expenditure reduces the chances of getting a job probably because women might not need to work to cover costs associated to healthcare. Then again, once in the job market, women are more inclined to work given increased expenditures on healthcare.

Table 3 below presents the result for the sample of *men*. In this table, we take into consideration a potential selection bias of the working men population, even though 80 % of men do work. Results are very fascinating in the sense that, men's labor supply is positively and consistently affected by wage (w – see columns 1 to 15). Income with transfers (T which is statistically significant in all specifications) and virtual income (Y) reduce the number of hours worked of men (columns 1 to 15 except *single men* having virtual income with non-significant and positive effects). The non-appropriated (G) presents significant negative signal for the entire sample but positive for single *men* sample. For disaggregated public expenditures (G - housing, education and health), we observe that healthcare expenditure negatively, but habitation and education positively, affects the number of hours worked consistently. It seems that expenditures do not produce robust effects (education negatively affect *married* and positively affect *single* and the aggregated G is not significant) with regard to the decision to enter into the job market.

**Table 3 - Summary of 2-stage regression for Men (corrected selection bias)**
(w, T, Y and the decision to work as endogenous)

| | Men | | | | | | Married Men | | | | Single Men | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Dep: Hours/Month | no G, T | no G | no T | Total State Public Expenditure | Public State Expenditure - Parts | no G, T | no G | no T | Total State Public Expenditure | Public State Expenditure - Parts | no G, T | no G | no T | Total State Public Expenditure | Public State Expenditure - Parts |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep | coef/ep |
| w | 3.132*** | 9.889*** | 1.712*** | 12.625*** | 7.550*** | 4.920*** | 2.448** | 7.179*** | 5.061*** | 3.804*** | -2.461*** | 1.244 | -2.415*** | -0.327 | 3.981*** |
| | (0.676) | (0.882) | (0.585) | (1.159) | (0.495) | (0.884) | (0.985) | (0.886) | (0.990) | (1.058) | (0.793) | (1.382) | (0.846) | (1.666) | (0.838) |
| T | | -0.103*** | | -0.134*** | -0.085*** | | -0.054*** | | -0.054*** | -0.072*** | | -0.036*** | | -0.020 | -0.053*** |
| | | (0.009) | | (0.012) | (0.006) | | (0.015) | | (0.014) | (0.013) | | (0.011) | | (0.014) | (0.009) |
| Y | -0.024*** | -0.087*** | -0.019*** | -0.112*** | -0.068*** | -0.039*** | -0.016 | -0.070*** | -0.050*** | -0.040*** | 0.029*** | -0.003 | 0.010 | -0.007 | -0.043*** |
| | (0.007) | (0.009) | (0.006) | (0.010) | (0.004) | (0.010) | (0.010) | (0.008) | (0.010) | (0.010) | (0.010) | (0.014) | (0.010) | (0.015) | (0.007) |
| G | | | 0.001*** | -0.001*** | | | | -0.000 | -0.000 | | | | 0.002*** | 0.002*** | |
| | | | (0.000) | (0.000) | | | | (0.000) | (0.000) | | | | (0.000) | (0.000) | |
| G - Health | | | | | -0.063*** | | | | | -0.022*** | | | | | -0.099*** |
| | | | | | (0.004) | | | | | (0.004) | | | | | (0.016) |
| G - Housing | | | | | 0.035*** | | | | | 0.006* | | | | | 0.079*** |
| | | | | | (0.003) | | | | | (0.003) | | | | | (0.012) |
| G - Education | | | | | 0.002*** | | | | | 0.000 | | | | | 0.003*** |
| | | | | | (0.000) | | | | | (0.000) | | | | | (0.001) |
| G - Remainder | | | | | 0.002*** | | | | | 0.002*** | | | | | 0.004*** |
| | | | | | (0.000) | | | | | (0.000) | | | | | (0.001) |
| **Probit - reduced form** | | | | | | | | | | | | | | | |
| G | | | -0.000 | -0.000 | | | | -0.000 | -0.000 | | | | 0.000 | 0.000 | |
| | | | (0.000) | (0.000) | | | | (0.000) | (0.000) | | | | (0.000) | (0.000) | |
| G - Health | | | | | 0.000 | | | | | 0.001*** | | | | | -0.001 |
| | | | | | (0.000) | | | | | (0.000) | | | | | (0.000) |
| G - Housing | | | | | -0.000*** | | | | | -0.001*** | | | | | -0.000 |
| | | | | | (0.000) | | | | | (0.000) | | | | | (0.000) |
| G - Education | | | | | 0.000 | | | | | -0.000 | | | | | 0.000*** |
| | | | | | (0.000) | | | | | (0.000) | | | | | (0.000) |
| G - Remainder | | | | | -0.000 | | | | | -0.000** | | | | | 0.000 |
| | | | | | (0.000) | | | | | (0.000) | | | | | (0.000) |
| N (2 stage) | 48,207 | 48,207 | | 48,207 | 48,207 | 34,254 | 34,254 | | 34,254 | 34,254 | 13,953 | 13,953 | | 13,953 | 13,953 |

note: 0.01 - *** ; 0.05 - ** ; 0.1 - * ;
standard errors in parenthesis

Briefly, transfers (T) negatively affect labor supply in the case of *women* (except for married) and aggregated public expenditures (G) are substitutes of leisure, that is, increased G reduces leisure (increased labor supply). In the case of *men*, transfers (T) are unambiguously negative on labor supply, but aggregated expenditure (G) does not robustly affect subsamples. Transfers are not significant for married, but it is negative and significant for the entire sample of *men* and positive and significant for single *men* sample.

Lastly, it is worth noting that transfers are crucial when determining labor supply and consequently when MSCKT is estimated. That is so, not only because it is significant but also because it affects the signal and the magnitude of other variables (columns 3, 8, 13, and 18 of Tables 2A and 2B, and columns 3, 8 and 13 of Table 3).

**4. Brazilian Data Simulation**

To conduct illustrative estimations based on a more detailed tax structure, we can extend the model described in Section 2 to capture the different preferences for the families of the sample. We follow Browning (1976), and assume that governmental expenditures are financed by an income tax. Let $l^h$ be the labor supply of family h, $\left(x_0^h,...,x_n^h\right)$ its consumption vector, $\tau^h$ the marginal tax rate for h and $\overline{y}$ the nontaxable income. Families face the same income and price vector $\left(w, p_1,..., p_n\right)$ and can have different incomes if their respective labor supply differs.

Usually, the marginal tax rate of a family depends on its income level. However, when marginal changes are considered around an initial equilibrium, we can assume that its marginal tax rate is not affected by small variations of the family income. In other words, no family starts with income at the dividing point between two income levels. Similarly, the budget constrain of family h is $\sum_{i=0}^{n} p_i x_i^h = wl^h - \tau^h (wl^h - \overline{y})$ which leads to the budget constraint for the government $\sum_h \tau^h (wl^h - \overline{y}) = c_1(G_1) + c_2(G_2) = c(G_1, G_2)$.

Additionally, let $v_Y^h = \partial v^h / \partial Y^h$ be the marginal utility of virtual income for family h. Now consider Roy's identity ($(\partial v^h / \partial \tau^h) / v_Y^h = -wl^h + \overline{y}$) and Slutsky's equation ($\partial l^h / \partial \tau^h = -w(\partial l^h / \partial \overline{w^h}) + \overline{y}(\partial l^h / \partial Y^h)$) for this model. In which $\overline{w}^h = \left(1 - \tau^h\right) w$ is the net (of tax) wage, $v^h$ is the family's indirect utility function. As in Wildasin (1984), we simply hypothesize that elasticity (ordinary income of labor supply $\left(\varepsilon_{l\overline{w}}\right)$) is the same for all families. The Slutsky's equation implies that $\overline{w}\left(\partial l^h / \partial I^h\right)$, the total income elasticity of labor supply is also the same for all h. Once again, we following Wildasin (1984) and consider three types of taxes:

a) proportional ($\tau^h = \tau$ for any h; $\bar{y} = 0$);

b) linear progressive or regressive ($\tau^h = \tau$ for any h; $\bar{y} = 0$);

c) non-linear progressive (unequal $\tau^h$; $\bar{y} = 0$).

We want to evaluate the marginal change in the quantity of public Goods $G_1$ and $G_2$ followed by a change in the tax rate which maintains the government budget under balance. In cases of proportional and linear progressive taxes we use the budget constraint of the government to implicitly resolve for $\tau$ in terms of $G_1$ and $G_2$. In case of non-linear progressive tax, we follow Browing's assumption that all marginal tax rates are proportionally progressive and, once again, resolve for changes in $\tau^h$ using that budget constraint.

In order to obtain welfare indicators for evaluating G, we use a welfare function W (Bergson-Samuelson) that satisfies "simple neutrality" (SN), that is, the social marginal utilities of income are the same. That is, for a given μ, $(\partial W / \partial v^h) v_Y^h = \mu, \textit{for all } h$. To evaluate changes in G, we first assume a proportional tax and when W is totally differentiated and divided by μ, we have:

$$\frac{1}{\mu}\frac{dW}{dG} = \alpha \sum_h MRS_1^h + (1-\alpha)\sum_h MRS_2^h - \frac{\alpha\left[MRT_1 - \tau w \sum_h \partial l^h \big/ \partial G_1\right] + (1-\alpha)\left[MRT_2 - \tau w \sum_h \partial l^h \big/ \partial G_2\right]}{1 - \frac{\tau}{1-\tau}\varepsilon_{l\bar{w}}} \tag{4}$$

With a linear progressive tax ($\tau^h = \tau, \bar{y} \neq 0$) the result is:

$$\frac{1}{\mu}\frac{dW}{dG} = \alpha \sum_h MRS_1^h + (1-\alpha)\sum_h MRS_2^h - \frac{\sum_h (wl^h - \bar{y})\left[\alpha\left[MRT_1 - \tau w \sum_h \partial l^h \big/ \partial G_1\right] + (1-\alpha)\left[MRT_2 - \tau w \sum_h \partial l^h \big/ \partial G_2\right]\right]}{\sum_h wl^h\left(1 - \frac{\tau}{1-\tau}\varepsilon_{l\bar{w}}\right) - \sum_h \bar{y}\left(1 - \frac{\tau}{1-\tau}\bar{w}\frac{\partial l^h}{\partial Y^h}\right)} \tag{5}$$

With a non-linear progressive tax (unequal $\tau^h$; $\bar{y} \neq 0$), we obtain:

$$\frac{1}{\mu}\frac{dW}{dG} = \alpha\sum_h MRS_1^h + (1-\alpha)\sum_h MRS_2^h - \frac{\sum_h (wl^h - \bar{y})\left[\alpha\left[MRT_1 - \tau^h w\sum_h \partial l^h \big/ \partial G_1\right] + (1-\alpha)\left[MRT_2 - \tau^h w \sum_h \partial l^h \big/ \partial G_2\right]\right]}{\sum_h \tau^h \left[wl^h\left(1 - \frac{\tau}{1-\tau}\varepsilon_{l\bar{w}}\right) - \bar{y}\left(1 - \frac{\tau}{1-\tau}\bar{w}\frac{\partial l^h}{\partial Y^h}\right)\right]} \qquad (6)$$

Note that equation (6) is more general, of which (5) and (6) are special cases. Formula (7) is equivalent to equation (3) of the simple consumer model. Equations (2), (4), (5), and (6) of MSCKT show that we need labor supply elasticity in relation to wage ($\varepsilon_{l\bar{w}}$), and the response of labor supply to variations of public expenditure ($\partial l^h / \partial G_1$ *and* $\partial l^h / \partial G_2$ )).

With formulas (4) to (6) in hand, we can estimate the marginal cost of public funding for Brazil, with or without the ordinary independence between labor supply and the publicly provided Goods. The bundle of goods considered for this analysis is a combination of in-kind Goods ($G_1$ = total government spending minus transfers) and cash transfers from the government to citizens ($G_2$ = total spending with direct transfers such as pension, *Bolsa-Familia*, etc.). Given that in-kind goods are defined in terms of numeraire, the marginal rate of transformation is equal to one for $G_1$ and for $G_2$. We are using the results obtained for the men sample (see Table 3, column 4). The estimated value for the ordinary elasticity of labor supply with respect net wage is 0.41 and for the total income elasticity is -0.18. The average wage for men is R$5.7 per hour and they work in average 172 hours per month. In cases of proportional and linear progressive taxes we use $\tau$ = 0.373 given that the tax burden is around 37.37% of the GDP. For the non-linear progressive tax we use a weighted average of the three types of taxes: 0% (between R$0 and R$1,257.12), 15% (between R$1,257.13 and R$2512.08), and 27.5% (> R$2512.09).[9]

[9]The 10% richest subsample has an average income of R$50,000. Each individual pays 0x1,257.12 + 0.15x(2,512.08 – 1,257.12) + 0.275x(50000 – 2,512.08) = R$1,3247.45 which results in a effective aliquot of 26% approximately. Those individuals appropriate 45% of the GDP and the next 30% (a total of 40% of individuals paying income tax) appropriates 34% of the GDP. In the case of the non-linear progressive tax, the collected income is then 0.34x0.15 + 0.45x0.26 = 0.168 of the taxable income.

When ordinary dependency is allowed between labor supply and the provision of this bundle of goods, we use -0.001h/R$ for ∂l/∂$G_1$ and -0.134 h/R$ for ∂l/∂$G_2$.Table 4 shows the values estimated of the marginal cost of cash-cum-in-kind transfers for proportional taxes where the population is normalized (Conway, 1997) when both ordinary independency and dependency (between labor supply and Goods $G_1$ and $G_2$) are allowed.[10]

**Table 4 - Marginal Cost of Cash-Cum-in-Kind Transfers**

| Marginal Costs | Bundle Goods Proportion | | Assumed Independency between Job and Public Goods | | | Assumed Ordinary Dependecy between Job and Public Goods | | |
|---|---|---|---|---|---|---|---|---|
| | alfa | 1-alfa | MSCKT1 | MSCKT2 | Total MSCKT | MSCKT1 | MSCKT2 | Total MSCKT |
| **Proportional Tax** | 0.950 | 0.050 | 1.258 | 0.066 | 1.324 | 1.258 | 0.085 | 1.343 |
| | 0.800 | 0.200 | 1.059 | 0.265 | 1.324 | 1.060 | 0.339 | 1.399 |
| | 0.700 | 0.300 | 0.927 | 0.397 | 1.324 | 0.927 | 0.509 | 1.436 |
| | 0.600 | 0.400 | 0.795 | 0.530 | 1.324 | 0.795 | 0.679 | 1.474 |
| | 0.500 | 0.500 | 0.662 | 0.662 | 1.324 | 0.662 | 0.849 | 1.511 |
| **Linear Progressive Tax** | 0.950 | 0.050 | 1.395 | 0.073 | 1.468 | 1.395 | 0.094 | 1.489 |
| | 0.800 | 0.200 | 1.174 | 0.294 | 1.468 | 1.175 | 0.376 | 1.551 |
| | 0.700 | 0.300 | 1.028 | 0.440 | 1.468 | 1.028 | 0.564 | 1.592 |
| | 0.600 | 0.400 | 0.881 | 0.587 | 1.468 | 0.881 | 0.753 | 1.634 |
| | 0.500 | 0.500 | 0.734 | 0.734 | 1.468 | 0.734 | 0.941 | 1.675 |
| **Non-linear Progressive Tax** | 0.950 | 0.050 | 1.134 | 0.060 | 1.193 | 1.134 | 0.067 | 1.201 |
| | 0.800 | 0.200 | 0.955 | 0.239 | 1.193 | 0.955 | 0.269 | 1.224 |
| | 0.700 | 0.300 | 0.835 | 0.358 | 1.193 | 0.835 | 0.404 | 1.239 |
| | 0.600 | 0.400 | 0.716 | 0.477 | 1.193 | 0.716 | 0.538 | 1.254 |
| | 0.500 | 0.500 | 0.597 | 0.597 | 1.193 | 0.597 | 0.673 | 1.270 |

It is worth highlighting the discrepancy among the values depending on the assumption regarding ordinary dependency versus independency. When there is independency, total MSCKT is 1.32 a value roughly close to the usual empirical results described in the literature. In other words, for each Real (R$) collected by the government through tax $t_1$, the costs incurred by society is R$1.32. Obviously, with α = 0.5, $G_1$'s MSCKT is the same as $G_2$'s counterpart.

On the other hand, for proportional tax, when ordinary dependency is assumed between labor supply and public Goods $G_1$ and $G_2$, total MSCKT varies between R$ 1.34 R$ and R$ 1.51 depending on the value of α. When α = 0.5, MSCKT of public Good 1 is 0.662 and of public Good 2 is 0.85. That is so

[10] When using the equations above, it should be noted that the income tax is not the only source of tax revenues in Brazil. If the rates used are applied only to labor income, it would provide a lesser amount of revenues than if applied to the total revenue – that would imply smaller government revenues than the observed. To compensate for this, assume the existence of a lump-sum tax that, along with the income tax, brings the total tax revenue up to the total observed, but it is not used for funding any increments in public expenditure. It is important to stress that the hypothesis about independency (ordinary and compensated, omitted in this version) are not compatible, therefore, to assume the presence of one is to admit the absence of the other and those are only extreme theoretical cases.

because cash transfers ($G_2$) have a greater effect (more negative) on labor supply than government spending in publicly provided Goods ($G_1$). Note that Conway (1997) found MSCKT values between 1.04 and 1.69 for US - the highest value happens when redistributive spending is considered. For linear progressive taxes, our sample has an exemption rate equals to 10.6% of total income (for men this is calculated by dividing exempt income by total income, $\sum_h \bar{y} = 0{,}106 \sum_h wl^h$ ) and the resulting MSCKT are displayed in Table 4, rows 6-10.

Table 4 also shows that with ordinary independency and a linear progressive tax, MSCKT is 1.47, greater than the proportional tax (1.32) – implying that MSCKT also depends on the tax system as observed by Wildasin (1984). This result is maintained with ordinary dependency; however, total MSCKT ranges between 1.49 and 1.67 above the MSCKT interval of 1.342 to 1.511 for the proportional tax. In the case that 50% of the public expenditures go to income transfers (T) we observe MSCKT 14% larger than the case that independency ordinary is assumed. In the case of non-linear progressive tax (Table 4, rows 11-15), we have that total MSCKT with ordinary independency is 1.19, below the previous results. This value (below in comparison to other taxes) are kept even when ordinary dependency is assumed with a MSCKT varying between 1.20 and 1.27 (an increase up to 6% compared to the ordinary independency situation). However, it is worth noting that, we use the effective tax rate of 16.8%. In order to compare the three tax systems we impose a tax rate of 16.8% as shown by Table 5.

Table 5 reinforces that MSCKT changes according to the tax system considered, and suggests that the non-linear progressive tax leads to the highest costs to society. The exemption rate also affects this outcome, specifically, the greater the exemption rate (exemption rate from 10% - calculated from our sample – or 21%- calculated from Brazilian Treasury data) the lesser the indirect cost (MSCKF) for society (lines 2 and 4; 3 and 5). Lastly, Table 5 shows that by including only aggregated expenditure ($G_1$), marginal social cost stay nearly unaltered (compared to ordinary independency) given that the response in terms of labor supply (taxed good) of agents is not too sensitive to this expenditure (see columns 1 and 3). However, when we allow ordinary dependency of labor supply and transfers (T) we observe significant upward alterations in the estimation of MSCKF. That is the case even when only 5% of expenditure is composed by transfers. Moreover, the effect is 4% above the estimated when only $G_1$ is considered (column 3). This effect intensifies when the proportion between public spending and direct transfers increases.

**Table 5 - MSCKT for Three Types of Tax Systems (different exemption rate)**

| | Ordinary | | |
|---|---|---|---|
| | Independency | Dependency G and T | Dependency G |
| α = 0,95 | (1) | (2) | (3) |
| Proportional Tax<br>TMI = 0,168 | 1.091 | 1.099 | 1.092 |
| Linear Progressive Tax<br>TMI = 0,168 TI = 0,106 | 1.123 | 1.130 | 1.123 |
| Non-linear Progressive Tax<br>TMI = 0,168 TI = 0,106 | 1.193 | 1.200 | 1.193 |
| Linear Progressive Tax<br>TMI = 0,168 TI = 0,21 | 1.163 | 1.171 | 1.163 |
| Non-linear Progressive Tax<br>TMI = 0,168 TI = 0,21 | 1.262 | 1.270 | 1.263 |

## 5. Conclusion and discussion

The main goal of this work is to estimate the marginal social costs of publicly provided goods when taxpayers are distortedly taxed but can receive subsidies when consuming that public good. This scenario allows precise characterization of the cost-benefit relation of transfers programs and has policy implications. We also estimate individuals' response to this bundle of goods (monetary and non-monetary) in terms of labor supply and conduct computations of marginal costs of public funding in Brazil.

Our estimations suggest that we should take into consideration the composition of public expenditure (in kind versus cash transfers) in order to determine the marginal cost of public fund. To the best of our knowledge this paper is the first to explore the response of labor supply to variations in public expenditures along with transfers received by families at the same time as MSCKT computed. Our simulations suggest that MSCKT increases up to 14% if compared to cases in which cash-cum-in-kind transfers is an ignored public policy. Moreover, most of this effect comes from the negative impact of cash transfer on labor supply response on the part of individuals.

Further panel data experiments based on municipal public finance data should be conducted in order to circumvent the agents' heterogeneity problem inherent in cross section analysis – and individuals' labor supply response could be more sensitive at this data level. Last, such cost-benefit analysis make more sense when a specific project is considered and therefore its effects on the taxed good can be clearly estimated leading to a more reliable estimative of the marginal social cost of funding that project.

**References**


Atkinson, A.B. and Stern, N. (1974), Pigou, Taxation and Public Goods. *Review of Economic Studies*, 41, pp. 119—128.

Avelino,R. and Menezes-Filho, N. (2003) Estimação da Oferta de Trabalho das Mulheres no Brasil. *Estudos Econômicos*, v.33, n. 4, pp. 639-664.

Ballard, C., (1990), Marginal Welfare Cost Calculations: Differential Analysis vs. Balanced Budget Analysis. *Journal of Public Economics*, 41, pp. 263—273.

Ballard, C. and Fullerton, D. (1992), Distortionary taxes and the provision of public goods. *Journal of Economic Perspectives*, 6, pp. 117—131.

Ballard, Shoven and Whalley (1985), General Equilibrium computations of the marginal welfare costs of taxes in the United States. *American Economic Review*, 75, pp. 128--138.

Blundell, Richard and MaCurdy Thomas. (1999). Labor Supply: a Review of Alternative Approaches in: Blundell, R., A.Duncan and C. Meghir (1998). Estimating Labour Supply Responses to Tax Reform. *Econometrica* 66, 827--861.

Browning, E. (1987), On the marginal welfare cost of taxation. *American Economic Review*, 77, pp. 11--23.

Burtless, G. & Hausman J. A. (Dec., 1978). "The Effect of Taxation on Labor Supply: Evaluating the Gary Negative Income Tax Experiment". *The Journal of Political Economy*, Vol. 86, No. 6, pp. 1103-1130.

Conway, K. (1997), Labor supply, taxes, and government spending: a microeconometric analysis. *Review of Economics and Statistics*, 79, pp. 50—67.

Conway, K. and Kriesner, T.(1992), How fragile are male labor supply function estimates?. *Empirical Economics*, 17, pp. 169—182.

DaVanzo, J. e Tray, D. N. e Greenberg, D. H. (Agosto, 1976). "The Sensitivity of Male Labor Supply Estimates to Choice of Assumptions". *The Review of Economics and Statistics*, Vol. 58, No. 3, pp. 313-325

Deaton, Angus. (1997), The Analysis of Household Surveys. A Microeconometric Approach to Development Policy. Baltimore: John Hopkins University Press for the World bank.

Diamond, P. A. e McFadden, D. L., "Some Uses of the Expenditure Function in Public Finance", *Journal of Public Economics* 3, 1974, 3-21.

Fullerton, D. (1991), Reconciling recent estimates of the marginal welfare cost of taxation. American Economic Review, 81, pp. 302—308.

Gahvari, F. (1986), Labor supply and tax rates: Comment. *American Economic Review*, 76, pp. 280—283.

Gahvari, F. (2006), On the marginal cost of public funds and the optimal provision of public goods, *Journal of Public Economics*, forthcoming.

Gahvari, F. and Mattos, (2007). Conditional cash transfer, public provision of private good and income redistribution, *American Economic Review*, v. 97, p.491-502.

Gwartney, J. and Stroup, R. (1983), Labor supply and tax rates: A correction of the record. *American Economic Review*, 73, pp. 446—451.

Harberger, A. (1964), The measurement of Waste. *American Economic Review*, 54, pp. 58—76.

Heckman , J. (1976). The common structure of statistical models of truncation, sample selection, and limited dependent variables and a simple estimator of such models. *Annals of economic and social measurement*, 5, 475-492.

Heckman, J. J. (May, 1993). "What Has Been Learned About Labor Supply in the Past Twenty Years?" *The American Economic Review*, Vol. 83, No. 2, Papers and Proceedings of the *Hundred and Fifth Annual Meeting of the American Economic Association*, pp. 116-121.

Lundberg, S. (May, 1988). "Labor Supply of Husbands and Wives: A Simultaneous Equations Approach". *The Review of Economics and Statistics*, Vol. 70, No. 2, pp. 224-235.

MaCurdy, T. Green, D. and Paarsch, H. (1990), Assessing Empirical approaches for analyzing taxes and labor supply. *Journal of Human Resources*, 25, pp. 415—490.

Mattos, E. (2007), Cash-cum-in-kind transfer, tax rates and labor supply: Qualifying the record, Anpec – 2007.

Pigou, A. (1947), A study in Public Finance, 3a. edition, London: Macmillan.

Resende, A. C. C. & Oliveira, A. M. H. C. (2006). "Avaliando resultados de um programa de transferência de renda: O impacto do Bolsa-Escola sobre os gastos das famílias brasileiras". Anpec.

Samuelson, P. (1954), the pure theory of public expenditure. *Review of Economics and Statistics*, 36, pp. 387—389.

Snow, A. e Warren, R. (1989), Tax rates and labor supply in fiscal equilibrium. *Economic Inquiry*, 27, pp. 511—520.

Slemrod, J. and Yitzhaki, S., "Integrating Expenditure and Tax Decisions: The Marginal Cost of Funds and the Marginal Benefit of Projects", *National Tax Journal* 54, 2001, 189-200.

Stiglitz, J. and Dasgupta, P. (1971), Differential taxation, public goods and economic efficiency. *Review of Economic Studies*, 38, pp. 151—174.

Stuart, C. (1984), Welfare costs per dollar of additional tax revenue n the United States. *American Economic Review*, 74, pp. 352—362.

Triest, R.(1987). "A Monte Carlo test of the robustness of alternative estimators of labor supply functions", Working Paper (198), Johns Hopkins University.

Wildasin, D. (1984), On public good provision with distortionary taxation. *Economic Inquiry*, 22, pp. 227-243.

Wilson, J. (1991), Optimal public good provision with limited lump-sum taxation. *American Economic Review*, 81, pp. 153—166.

**Appendix**

**Robustness – Nonlinearity among G, T and labor supply**

Table 4 shows estimations that take into consideration the fact that aggregated government expenditure and transfers can directly affect individuals' response of labor supply consequently affecting other independent variables. These specifications consider the following regression:

$$h_i = \beta w_i + \gamma Y_i + \varphi T_i + \phi G + \delta w_i T_i + \eta T_i G + \lambda Y_i T_i + \tau w_i G + \rho Y_i G + X\Gamma + \varepsilon$$

(1A)

where $wT_i$, $YT_i$ and $T_iG$ are interactions among the variables wage/hour (w), virtual income (Y) and the aggregated public expenditure (G) with income transfer (T) and *wG*, *YG* and *TG* are interactions among the variables wage/hour (w), virtual income (Y) and income transfer (T) with the aggregated public expenditure (G). The marginal effect shows that wage positively affects *men* (except for *single men*) but not for *single* and *married women* women as to the number of hours worked. Virtual income (Y) negatively affects all *women* subsamples (except married). Transfers (T) negatively affects all subsamples as shown in Conway (1997) who shows that distributive expenditure negatively affect labor supply. And the aggregated expenditure (G) positively affects labor supply in all subsamples. In relation to the statistic significance, we conduct a F-test where the null hypothesis states that all marginal components are non-significant and the alternative says that it is at least significant. Aggregated expenditure (G) and appropriated (T) are significant in all samples, except for *married men* which only income transfers (T) expenditure is significant. The results of this regression are displayed in the Table A1 below.

**Table A1- Summary of non-linear regression (corrected self-selection bias)**
(w, T, Y and the decision to work as endogenous)

[redacted]

| | 2 Stages - selection bias corrected | | | 2 Stages - selection bias corrected | | |
|---|---|---|---|---|---|---|
| Dep: Hours/Month | **men** | **married men** | **single men** | **women** | **married women** | **single women** |
| | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** | **(6)** |
| | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** | **coef/ep** |
| w | 11.513*** | 5.125*** | 1.452 | 7.255*** | -4.881*** | -1.300 |
| | (1.101) | (1.188) | (1.722) | (1.413) | (1.790) | (1.406) |
| T | -0.173*** | -0.082*** | -0.068*** | -0.124*** | -0.049** | -0.033** |
| | (0.011) | (0.015) | (0.017) | (0.014) | (0.021) | (0.014) |
| Y | -0.096*** | -0.051*** | -0.024 | -0.031*** | 0.032*** | 0.008 |
| | (0.010) | (0.012) | (0.016) | (0.007) | (0.008) | (0.012) |
| G | -0.001** | 0.000 | 0.002*** | 0.006*** | 0.007*** | 0.003*** |
| | (0.000) | (0.000) | (0.001) | (0.001) | (0.001) | (0.001) |
| δ | 0.004*** | -0.005** | 0.002*** | 0.003*** | 0.009*** | 0.004*** |
| | (0.001) | (0.002) | (0.001) | (0.001) | (0.002) | (0.001) |
| λ | -0.000*** | 0.000*** | 0.000 | 0.000*** | -0.000*** | -0.000*** |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| τ | 0.000 | -0.000 | -0.000*** | -0.001*** | -0.000 | 0.000 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| ρ | -0.000*** | -0.000 | -0.000 | 0.000*** | -0.000** | -0.000 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| η | 0.000*** | 0.000 | 0.000*** | 0.000 | 0.000*** | -0.000 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| Derivatives | | | | | | |
| $\frac{\partial h}{\partial w} = \beta + \delta T + \tau G$ | 11.513 | 5.125 | -1.560 | 7.255 | -4.881 | -6.194 |
| $\frac{\partial h}{\partial Y} = \gamma + \lambda T + \rho G$ | -0.096 | -0.051 | -0.024 | -0.031 | 0.032 | 0.008 |
| $\frac{\partial h}{\partial T} = \varphi + \delta w + \eta G + \lambda Y$ | -0.173 | -0.082 | -0.068 | -0.124 | -0.049 | -0.033 |
| $\frac{\partial h}{\partial G} = \phi + \eta T + \tau w + \rho Y$ | -0.001 | 0.000 | 0.002 | 0.006 | 0.007 | 0.003 |
| Tests | | | | | | |
| [redacted] | 51.39*** | 4.36 | 44.1*** | 162.97*** | 147.78*** | 26.05*** |
| [redacted] | 264.4*** | 42.58*** | 33.8*** | 100.19*** | 67.18*** | 21.32*** |
| adjusted R2 | . | . | . | . | | |
| N | 48,207 | 34,254 | 13,953 | 72,542 | 48,611 | 23,931 |

note: 0.01 - ***; 0.05 - **; 0.1 - *;
standard errors in parenthesis