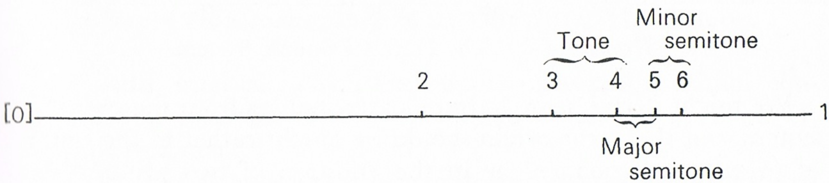
CHAPTER FIVE

On the origin of Dissonances and on their relationships

Dissonances[41] can be derived from the same divisions of the string that produced consonances, by comparing together the lengths, taken to the left, which remain after each number.[42] [Ex. I.5.] This also reveals the difference between two consecutive consonances. For example, the lengths taken to the left after the numbers 3 and 4 will give the tone, which differentiates the fifth from the fourth; those of the numbers 4 and 5 will give the major semitone, which differentiates the fourth from the major third; and those of the numbers 5 and 6 will give the minor semitone, which differentiates the major third from the minor third. These are the tones and semitones which form the successive degrees of the natural voice, from which melody originates. We begin to perceive, therefore, that melody is only a consequence of harmony.



Example I.5

The ratios of these dissonances can be learned by a rule of subtraction,[43] placing one above the other the ratios of two consecutive consonances whose difference is sought.[44] [Ex. I.6.]

[41] See the Alphabetic Table [Table of Terms]. [R.]

[42] This is the same diagram as Example 4; it is explained in footnote 27. As in Example 4, if [o]–1 is *C′*, then [o]–6 is *E♭′* and [o]–5 is *E′*. Hence, the difference between [o]–6 and [o]–5 is a minor semitone, etc. [P.G.]

[43] An annotation here in the Opéra copy changes this word to "multiplication." This change is simply incorrect. [P.G.]

[44] The ratio of the interval which is the sum of two intervals can be found by multiplying the ratios of the latter two intervals. The ratio of the interval which is the difference of two intervals can be found by dividing the ratio of the larger by the ratio of the smaller. Thus, the difference between the fifth and the fourth is $\frac{2}{3} \div \frac{3}{4}$ or $\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$, the interval of a tone. It is this latter operation that Rameau refers to as a "rule of subtraction." [P.G.]

Ratio of the fifth	2:3
Ratio of the fourth	3:4
Product	8:9

Example I.6

The cross ╳ signifies that the antecedent of one ratio must be multiplied by the consequent of the other; thus, $2 \times 4 = 8$ and $3 \times 3 = 9$. This product 8:9 gives us the ratio of the tone. We may do the same with the fourth and the major third, etc. If we seek the difference between the fifth and the major sixth, however, we shall find that it is a tone whose ratio is 9:10. This obliges us to distinguish two sorts of tones, calling the first major and the second minor.

On the basis of these observations, the following system has been established. [Ex. I.7.]

There is a:	*from:*	*as:*
minor tone	Do to Re	9:10
major tone	Re to Mi	8:9
major semitone	Mi to Fa	15:16
major tone	Fa to Sol	8:9
minor tone	Sol to La	9:10
major tone	La to Si	8:9
major semitone	Si to Do	15:16

Example I.7. The Perfect Diatonic System.

We might derive some harmonic dissonances from the preceding system, but their true origin should be sought rather in the squares of primary consonances or in the addition of two primary consonances, as the following demonstration shows.[45] [Ex. I.8.]

The other dissonances arise from the inversion of these latter. For example, the second arises from the seventh, the tritone from the false fifth, and the augmented second from the diminished seventh. Dissonances, such as the diminished second and the diminished fourth, arising from [the inversion of] augmented dissonances, have no place in harmony, for augmented dissonances are admitted only by supposition.[46] They may be found only together with the ninth or

[45] Rameau recognizes only three types of sevenths: diminished, perfect, and augmented. These are equivalent, respectively, to our modern diminished, minor, and major sevenths. This question is discussed in detail by Rameau in Book II, Chapter 29. [P.G.]

[46] This is the first time Rameau uses this term in the body of the *Traité*. It had a different, more general, meaning in eighteenth-century France (see Book III, Chapter 39). Although in Rameau's restrictive sense it might well be translated "sub-position" (see Joan Ferris, "The Evolution of Rameau's Harmonic Theories," in the *Journal of Music Theory* III (1959), p. 231), we have decided to retain the more general term here. [P.G.]

Addition of the ratio of the minor third to 5:6 that of the fifth 2:3 The product is the ratio of the seventh 10:18	Square of the ratio of the fourth 3:4 3:4 The product is the ratio of the same seventh 9:16
Addition of the ratio of the major third to 4:5 that of the fifth 2:3 The product is the ratio of the augmented seventh 8:15	Square of the ratio of the major third 4:5 4:5 The product is the ratio of the augmented fifth 16:25
Square of the ratio of the minor third 5:6 5:6 The product is the ratio of the false fifth 25:36	Square of the ratio of the fifth 2:3 2:3 The product is the ratio of the ninth 4:9
Cube of the ratio of the minor third 5:6 5:6 5:6 The product is the ratio of the diminished seventh 125:216	Addition of the ratio of the minor sixth to 5:8 that of the major sixth 3:5 The product is the ratio of the eleventh 15:40

Example I.8. Demonstration of the Origin of the Dissonances.

the eleventh, intervals which exceed the octave and cannot consequently be inverted. This is explained at length in Book II, Chapters 10 and 11.

Although we have said that harmonic dissonances may be formed only from primary consonances, we have nonetheless formed some of them from the fourth and the sixths. The eleventh given by the sixths, however, does not have the same privilege as the others; i.e.,

Fourth from Do to Fa		*Fourth from Re to Sol*	
From Do to Re: minor tone	9:10	From Re to Mi: major tone	8:9
From Re to Mi: major tone	8:9	From Mi to Fa: major semitone	15:16
Product	72:90	Product	120:144
From Mi to Fa: major semitone	15:16	From Fa to Sol: major tone	8:9
Product	1080:1440	Product	960:1296
Product reduced to lowest terms	3:4	Product reduced to lowest terms	20:27

Example I.9. Demonstration of the Two Different Ratios for the Interval of a Fourth.

Names of the Intervals generated first	*Natural ratios according to the divisions*	*Ratios altered by a comma*	*Names of the intervals which are inversions of the first*	*Natural ratios of the inverted intervals*	*Ratios of the inverted intervals altered by a comma*
Diminished comma	2025:2048	These first five intervals have only a single ratio and are never inverted. All the other intervals are constructed from their addition.			
Comma	80:81				
Minor diesis or enharmonic	125:128				
Major diesis	243:250				
Least semitone	625:648				
Minor semitone according to theory; augmented unison according to practice	24:25	Mean semitone which exceeds the minor semitone by a comma 128:135	Diminished octave	25:48	Diminished: 135:256
Major semitone according to theory; minor second according to practice	15:16	Maximum semitone which exceeds the major semitone by a comma 25:27	Augmented seventh or major seventh	8:15	
Major tone according to theory; second according to practice	8:9	Minor tone which has a comma less than the major tone 9:10	Seventh	9:16	Exceeding 5:9
Augmented tone; or diminished third	225:256	Exceeding: 125:144	Augmented Sixth	128:225	Diminished: 72:125
Augmented second	64:75	Diminished: 108:125	Diminished seventh	75:128	Exceeding: 125:216
Minor third	5:6	Diminished: 27:32	Major sixth	3:5	Exceeding: 16:27
Major third	4:5	Diminished: 81:100	Minor sixth	5:8	Exceeding: 50:81
Fifth	2:3	Diminished: 27:40	Fourth	3:4	Exceeding: 20:27
Augmented fifth	16:25	Diminished: 81:125	Diminished fourth	25:32	Exceeding: 125:162
False fifth	25:36	Diminished: 45:64	Tritone or Augmented fourth	18:25	Exceeding: 32:45

Example I.10. Natural and Altered Ratios for All the Intervals.