On a positive solution for (p, q)-Laplace equations with two parameters *

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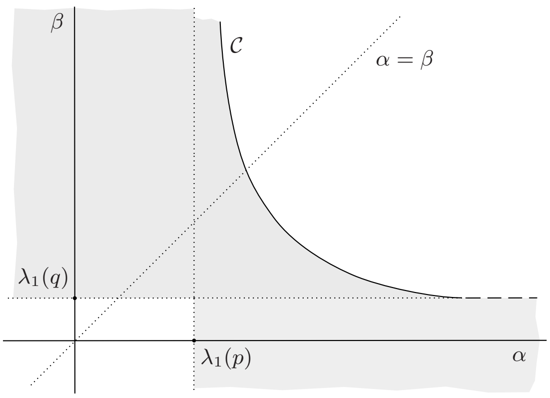
In this talk, I present the existence and non-existence of positive solutions for the (p, q)-Laplace equation $-\Delta_p u - \Delta_q u = \alpha|u|^{p-2}u + \beta|u|^{q-2}u$, where $p \neq q$, under the zero Dirichlet boundary condition in Ω. The main result is the construction of a continuous curve $\mathcal{C}$ in (α, β) plane, which becomes a threshold between the existence and non-existence of positive solutions.

Our result is deeply related to the first eigenvalues $\lambda_1(r)$ of the r-Laplacian Δ_r $(r = p,\ q)$. We denote the positive eigenfunction corresponding to $\lambda_1(r)$ by φ_r. Consider the following two cases:

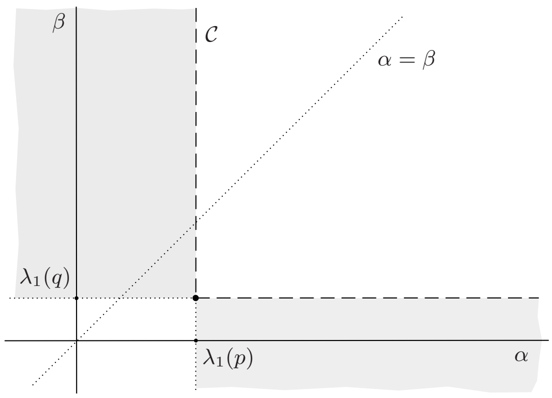
(i) $\lambda_1(p)$ and $\lambda_1(q)$ have different eigenspaces, namely,

(LI) For any $k \neq 0$ it holds $\varphi_p \not\equiv k\varphi_q$ in Ω.

(ii) $\lambda_1(p)$ and $\lambda_1(q)$ have the same eigenspace, namely, **(LI)** does not hold: $\varphi_p = k\varphi_q$ for some k



a) **(LI)** holds



b) **(LI)** is violated

Figure 1: Shaded sets correspond to existence, unshaded to non-existence

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