EXACT INTERIOR CONTROLLABILITY OF MAGNETOELASTIC PLATES BY MEANS OF PURELY MAGNETIC ACTUATION

Abstract. We consider a two-dimensional magnetoelastic plate coupled to two magnetic fields, with controls acting solely through the magnetic equations. Assuming that the constant directing magnetic vectors are linearly independent, we establish exact controllability between arbitrary finite-energy states in every prescribed time $T > 0$. The result covers both the presence and absence of rotational inertia.

The principal difficulty is the directional nature of the magnetoelastic coupling: each applied magnetic vector individually detects only one spatial direction and does not provide the coercivity required to recover the complete plate energy. The key ingredient is a reciprocal-basis identity that combines the two magnetic coupling operators and reconstructs the full Dirichlet Laplacian. This produces the complete kinetic-energy term in an operator-theoretic multiplier argument. Together with trace regularity and a compactness–uniqueness procedure, the identity yields the required observability inequality for the adjoint system. The result applies on general bounded, smooth, simply connected planar domains without special shape assumptions.

Let $\Omega \subset \mathbb{R}^2$ be bounded, simply connected and smooth, and let $H^{(1)}, H^{(2)} \in \mathbb{R}^2$ be linearly independent constant vectors. The controlled system is

$$\begin{gathered} w_{tt} - \gamma\Delta w_{tt} + \Delta^2 w - \sum_{j=1}^{2}\big(\operatorname{rot}\operatorname{rot} h^{(j)}\big)\cdot H^{(j)} = 0, \\ h_t^{(j)} + \operatorname{rot}\operatorname{rot} h^{(j)} + \operatorname{rot}\operatorname{rot}\big(H^{(j)} w_t\big) = \operatorname{rot}\bar{u}_j, \qquad j = 1,2. \end{gathered} \tag{1}$$

The plate is clamped, each magnetic state is divergence-free with vanishing normal component, and the homogeneous magnetic energy condition is imposed on the boundary $\partial\Omega$. The scalar control potentials satisfy $\bar{u}_j \in L^2(0,T;L^2(\Omega))$. Their curls are understood weakly through

$$\big\langle \operatorname{rot}\bar{u}_j, z\big\rangle = \big(\bar{u}_j, \operatorname{rot} z\big)_{L^2(\Omega)}, \qquad z \in \mathcal{V}_\sigma, \tag{2}$$

where $\mathcal{V}_\sigma$ is the magnetic form domain. Thus the controls are bulk L^2 potentials, while the corresponding magnetic forces belong to $\mathcal{V}_\sigma'$. In particular, no boundary trace of $\bar{u}_j$ is required, and the adjoint observation is $\big(\operatorname{rot}\psi^{(1)}, \operatorname{rot}\psi^{(2)}\big)$.

For every $T > 0$, $M > 0$, and $0 \le \gamma \le M$, system (1) is exactly controllable between arbitrary states in its natural finite-energy space by a pair $(\bar{u}_1, \bar{u}_2) \in L^2(0,T;L^2(\Omega))^2$. The observability constant, and hence the control cost furnished by the proof, is uniform for $\gamma \in [0, M]$.

To describe the central mechanism, let $\widehat{H}^{(1)}, \widehat{H}^{(2)}$ be the reciprocal vectors, characterized by

$$\widehat{H}^{(j)} \cdot H^{(k)} = \delta_{jk},$$

and define $G_H\xi = \operatorname{rot}\operatorname{rot}(H\xi)$ and $A_D = -\Delta$ with homogeneous Dirichlet boundary condition. Then

$$\sum_{j=1}^{2} \widehat{H}^{(j)} \cdot G_{H^{(j)}}\xi = A_D\xi. \tag{3}$$

For the adjoint magnetic variables, we use the scalar multiplier

$$q = A_D^{-1}\left(\widehat{H}^{(1)} \cdot \psi^{(1)} + \widehat{H}^{(2)} \cdot \psi^{(2)}\right).$$

Identity (3) ensures that combining the two adjoint magnetic equations makes the time derivative of this multiplier contain the complete plate velocity. Testing the adjoint plate equation with q therefore produces the full plate kinetic energy. The remaining magnetic terms are controlled by the two observed curls. This is the point at which linear independence of the applied vectors restores the coercivity needed by the scalar multiplier argument.

After estimating the boundary contribution by sharp plate trace regularity, one first obtains an observability estimate containing compact lower-order terms. A compactness–uniqueness argument removes those terms: vanishing of both magnetic observations, together with (3), reduces the remaining uniqueness question to the ordinary Dirichlet Laplacian. Hence

$$E_\gamma(T) \le C_{T,M,H} \sum_{j=1}^{2} \int_0^T \|\mathrm{rot}\,\psi^{(j)}(t)\|_{L^2(\Omega)}^2 \, dt, \tag{4}$$

which gives exact controllability by the Hilbert uniqueness method. The use of A_D^{-1} follows the indirect-control strategy for thermoelastic plates in [1], while the two-field magnetoelastic architecture is related to the stabilization framework of [2].

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