A STATIONARY PHASE METHOD AND RELATED DISPERSIVE ESTIMATES FOR A CLASS OF EVOLUTION EQUATIONS

Abstract. We derive dispersive-dissipative estimates for multipliers associated to a general class of wave-type equations, in particular for the viscous Boussinesq equation. The dispersion is related to the geometric hypotheses on the phase function and on the degeneracies that may happen at low and high frequencies. The dissipation interacts with the dispersion, influencing the decay rate of the solutions.

In this talk we study time-dependent estimates (in particular, dispersive estimates) to the initial value problems (IVP) to evolution equations in the form

$$\begin{cases} u_{tt} + 2\alpha(A)u_t + \beta(A)u = 0, \quad t > 0,\ x \in \mathbb{R}^n, \\ (u, u_t)(0, x) = (u_0, u_1)(x). \end{cases} \tag{1}$$

where A denotes a second-order homogeneous operator whose action is defined by

$$Af = \mathfrak{F}^{-1}(\rho(\xi)^2\, \hat{f}), f \in \mathcal{S}(\mathbb{R}^n),$$

with $\rho : \mathbb{R}^n \setminus \{0\} \to \mathbb{R}_+$ is a positively homogeneous smooth function of degree 1. We assume that the Hessian matrix $H_{\rho(\xi)}$ is positive semidefinite and has maximal rank for any $\xi \neq 0$.

In most cases, $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ and $\beta : \mathbb{R}_+ \to \mathbb{R}$ are polynomials. We are interested in the 'damped oscillations regime', i.e., $\alpha^2 < \beta$. In this case, the attrition α dampen the oscillations without cancelling them and the solution is

$$\hat{u}(t, \xi) = e^{-t\alpha(\rho(\xi)^2)} \left(\frac{\sin(t\omega(\rho(\xi))}{\omega(\rho(\xi))}\, \hat{u}_1(\xi) + \cos(t\omega(\rho(\xi)))\, \hat{u}_0 \right).$$

with $\omega(\rho(\xi)) = \sqrt{\beta(\rho^2) - \alpha^2(\rho^2)}$.

The considered model include the Klein-Gordon equation, the multidimensional Boussinesq and Improved Boussinesq equations, plate equations with or without mass and with or without rotational inertia.

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