ASYMPTOTIC STABILITY OF A TIMOSHENKO SYSTEM WITH LORD–SHULMAN HEAT CONDUCTION

Abstract. This work investigates the asymptotic behavior of a thermoelastic Timoshenko system under the Lord–Shulman heat conduction law acting on both the shear force and bending moment. We analyze how the localization of thermal dissipation affects the system's stability by examining the fully coupled framework alongside two single-dissipation sub-cases. For the fully coupled system, global well-posedness and exponential stability are established. For the single-dissipation scenarios, we prove that exponential stability is achieved if and only if specific stability number (χ_1 or χ_2) is equal to zero; otherwise, an optimal polynomial decay rate is obtained.

The Lord–Shulman Timoshenko System

We consider a beam of length $L > 0$ modeled by the coupled Timoshenko system with two temperature variations under the Lord–Shulman law in $(0, L) \times \mathbb{R}^+$:

$$\rho_1\varphi_{tt} - k(\varphi_x + \psi)_x + \kappa_1(\theta + \tau_0\theta_t)_x = 0 \tag{1}$$

$$\rho_2\psi_{tt} - b\,\psi_{xx} + k(\varphi_x + \psi) + \kappa_1(\theta + \tau_0\theta_t) + \kappa_2(\vartheta + \tau_1\vartheta_t)_x = 0 \tag{2}$$

$$\rho_3(\theta + \tau_0\theta_t)_t - \gamma_1\theta_{xx} + \kappa_1(\varphi_x + \psi)_t = 0 \tag{3}$$

$$\rho_4(\vartheta + \tau_1\vartheta_t)_t - \gamma_2\vartheta_{xx} + \kappa_2\psi_{xt} = 0 \tag{4}$$

where $\rho_i, k, b, \kappa_i, \gamma_i, \tau_i > 0$ are physical constants, and $(\varphi, \psi, \theta, \vartheta)$ represent the physical state variables. The system is subject to standard initial conditions and mixed Neumann–Dirichlet boundary conditions on $x \in \{0, L\}$.

Our main objective is to analyze how the localization of thermal dissipation affects the asymptotic behavior of the model. To achieve this, we investigate the fully coupled framework ($\kappa_1, \kappa_2 > 0$) alongside the two primary single-dissipation sub-cases: the system with dissipation acting only on the shear force ($\kappa_2 = 0$) and the system with dissipation acting only on the bending moment ($\kappa_1 = 0$). For each configuration, we establish global well-posedness and determine the exact conditions required to achieve exponential stability, as well as the conditions that lead to optimal polynomial energy decay rates.

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