EXTENSIBLE PLATE EQUATION WITH THERMAL COUPLING UNDER COLEMAN-GURTIN LAW AND EXPONENTIAL PERTURBATIONS

Abstract. This work is concerned with long-time dynamics of solutions of extensible plate equations with thermal coupling. The problem corresponds to a model of thermoelasticity based on a theory of Coleman-Gurtin heat flux. By considering the case where rotational inertia is present we show that the thermal dissipation is sufficient to stabilize the system under exponential nonlinear perturbations. We guarantee the existence of a finite-dimensional global attractor. In addition the existence of exponential attractors is also considered.

1. The problem

This work is concerned with long-time behavior of solutions for a class of semilinear problems related to thermoelastic extensible plates with Coleman-Gurtin heat law. More precisely, over a bounded domain $\Omega \subset \mathbb{R}^2$ with regular boundary Γ, we consider the following Initial Boundary Value Problem (IBVP):

$$u_{tt} + \Delta^2 u - M\left(\int_\Omega |\nabla u|^2 dx\right)\Delta u - \Delta u_{tt} + f(u) + \nu\Delta\theta = h \quad \text{in} \quad \Omega \times \mathbb{R}^+, \tag{1.1}$$

$$\theta_t - \omega\Delta\theta - (1-\omega)\int_0^\infty k(s)\Delta\theta(t-s)ds - \nu\Delta u_t = 0 \quad \text{in} \quad \Omega \times \mathbb{R}^+, \tag{1.2}$$

$$u = \Delta u = 0 \quad \text{on} \quad \Gamma \times \mathbb{R}^+ \quad \text{and} \quad \theta = 0 \quad \text{on} \quad \Gamma \times \mathbb{R}, \tag{1.3}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,t)|_{t\le 0} = \theta_0(x,-t), \quad x \in \Omega, \tag{1.4}$$

where $\theta_0(x,t)$ is the prescribed history for $t \le 0$.

Under proper assumptions on the functions M and f, in [2] the authors prove that the IBVP generates a dynamical system $S(t)$ in a proper extended phase space $\mathcal{H}$. More specifically, it is assumed a polynomial-type growth for f in order to control the nonlinearity perturbation in the system. Here, in the present work, our goal is to generalize such a condition by allowing exponential growth for f. Then, by means of the Trudinge-Moser estimate, we still prove the problem (1.1)-(1.4) generates a dynamical system and analyze its long-time dynamics.

2. Assumptions and Main Result

Let us assume that the following conditions:

(1) $M \in C^1(\mathbb{R})$ and there exist constants $\alpha \in [0, \lambda_1)$ and $c_M \ge 0$ such that

$$M(s)s \ge -\alpha s - c_M \quad \text{and} \quad \hat{M}(s) \ge -\alpha s - c_M, \;\; \forall\, s \ge 0, \tag{2.5}$$

where $\hat{M}(z) = \int_0^z M(s)\,ds$ and $\lambda_1 > 0$ be the first eigenvalue of $-\Delta$.

(2) $h \in L^2(\Omega)$ and $f \in C^1(\mathbb{R})$ is such that for every $\gamma > 0$, there exist $C_\gamma > 0$ satisfying

$$|f'(z)| \le C_\gamma e^{\gamma |z|^2}, \quad \forall\, z \in \mathbb{R}, \tag{2.6}$$

$$f(s)s \ge -\beta s^2 - c_f \quad \text{and} \quad \hat{f}(s) \ge -\frac{\beta}{2}s^2 - c_f, \;\; \forall\, s \in \mathbb{R}, \tag{2.7}$$

where $\hat{f}(z) = \int_0^z f(s)\, ds$, $c_f > 0$ and $\beta > 0$ is properly chosen.

Under the hypotheses (2.5)-(2.7), it is possible to prove that IBVP (1.1)-(1.4) is also well-posed on the same energy space $\mathcal{H}$, see e.g. [1], similar to what was done in [2]. Hence, the system also generates a dynamical system $(S(t), \mathcal{H})$ on a proper Hilbert space H.

Our main goal is to prove that $(\mathcal{H}, S(t))$ has the quasi-stability property, see e.g. the general theory raising from [3]. See also [4] for a more concise theory on quasi-stable dynamical systems. With such a property in hands, we are able to prove the following result.

Theorem 2.1. *Under the assumptions* (2.5)-(2.7)*, we have:*

- *The associated dynamical system $(\mathcal{H}, S(t))$ has a compact global attractor $\mathcal{A}$.*
- *The global attractor $\mathcal{A}$ is characterized by the unstable manifold emanating from the set of stationary solutions.*
- *The global attractor $\mathcal{A}$ has finite fractal dimension.*
- *$(\mathcal{H}, S(t))$ has a T-discrete exponential attractor $\mathcal{A}_{\exp}$ and $\mathcal{A} \subset \mathcal{A}_{\exp}$.*

Proof. The complete proof is given in [1]. □

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