INTERNAL EXACT CONTROLLABILITY OF BRESSE SYSTEM WITH MEMORY

ABSTRACT. In this work, we study the internal exact controllability in the Bresse system, adding a memory term in one of the equations. We further emphasize that such term could be inserted into any one of the equations, or even in two or three equations, and the results would be analogous except for the control time T. Our calculations show that a minimum time of control is necessary to guarantee the control efficiency. We also need a minimum region for the efficiency of such controls. The main result is obtained combining the Hilbert Uniqueness Method due to Lions (1) and compactness arguments.

In this work, we study the internal exact controllability in the Bresse system, adding a memory term in one of the equations.

$$\begin{cases} \rho_1\varphi_{tt} - k(\varphi_x + \psi + l\omega)_x - k_0 l(\omega_x - l\varphi) - k_0 l^2 \int_0^t M(\sigma, t)\varphi(\sigma) = f_1 \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + l\omega) = f_2 \\ \rho_1\omega_{tt} - k_0(\omega_x - l\varphi)_x + kl(\varphi_x + \psi + l\omega) = f_3 \end{cases} \tag{1}$$

in $Q = (0, L) \times (0, T)$. We assume Dirichlet conditions, i.e.,

$$\varphi(0, t) = \varphi(L, t) = \psi(0, t) = \psi(L, t) = \omega(0, t) = \omega(L, t) = 0, \tag{2}$$

for $t \in (0, T)$, and initial conditions

$$\begin{cases} \varphi(x, 0) = \varphi_0, \quad \varphi_t(x, 0) = \varphi_1, \\ \psi(x, 0) = \psi_0, \quad \psi_t(x, 0) = \psi_1, \\ \omega(x, 0) = \omega_0, \quad \omega_t(x, 0) = \omega_1, \end{cases} \tag{3}$$

for $x \in (0, L)$.

The problem of exact controllability in equations $(1) - (3)$ is formulated as follows:
Given $T > 0$, large enough, to find a Hilbert space $\mathcal{H}$ such that, for all initial data $\{\varphi_0, \varphi_1, \psi_0, \psi_1, \omega_0, \omega_1\} \in \mathcal{H}$ there are controls $f_1 = h_1(x, t)\chi, f_2 = h_2(x, t)\chi$ and $f_3 = h_3(x, t)\chi, h_1, h_2, h_3 \in L^2(l_1, l_2)$, where χ is the characteristic of $(l_1, l_2) \times (0, T)$ and $(l_1, l_2) \subset (0, L)$, so that the solution $\{\varphi, \psi, \omega\}$ of $(1) - (3)$ satisfies

$$\varphi(x, T) = \varphi_t(x, T) = \psi(x, T) = \psi_t(x, T) = \omega(x, T) = \omega_t(x, T) = 0. \tag{4}$$

In this work, we will use the technique employed by Yan (3), which gives us the exact controllability using the Hilbert Uniqueness Method (HUM) proposed by Lions in (1) and described again by Zuazua in (4), without proving the inverse inequality. We also needed to use Hömlgren´s Theorem. On the other hand, an inverse inequality for Bresse systems, proved by Soriano and Schulz in (2), will be applied.

For the results of observability and controllability, we will always consider $T > 2\alpha R$ where

$$\alpha := \max\{1, \frac{\rho_1}{k}, \frac{\rho_2}{b}, \frac{\rho_1}{k_0}\} \tag{5}$$

and

$$R := \max\{l_1, L - l_2\} \tag{6}$$

with $(l_1, L_2) \subset (0, L)$ being the range where the control mechanisms act.

Teorema : Given $T > 2\alpha R$ and $M(\sigma, t) \in C^2([0,\infty) \times [0,\infty))$, then the system $(1) - (3)$ is exactly controllable.

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