A STABILIZED MULTISCALE HYBRID-MIXED METHOD FOR REACTION-DOMINATED MODELS

Abstract. We propose the MHM-UNUSUAL (MHM-UN) method by extending the Unusual Stabilized Finite Element Method (UNUSUAL) to variable coefficients and Neumann boundary conditions, and embedding it into the local solvers of the Multiscale Hybrid-Mixed framework. We present a convergence analysis of the proposed method, along with its numerical validation.

The Multiscale Hybrid-Mixed (MHM) method [1] is a strategy that decomposes the exact solution of a boundary value problem into a global problem defined on the skeleton of a coarse mesh and independent local problems within each coarse element. For singularly perturbed problems, standard local Galerkin solvers often require very fine meshes to ensure accuracy. To overcome this limitation, we replace the Galerkin local solver with a stabilized formulation, namely the UNUSUAL method [2]. We first extend UNUSUAL to reaction–diffusion problems with variable coefficients and mixed boundary conditions, establishing well-posedness, proving convergence, and demonstrating through numerical tests its improved accuracy over Galerkin in singularly perturbed regimes. We then integrate this stabilized formulation into the local solvers of MHM, resulting in the MHM-UN method. The proposed approach is proven to be well-posed and convergent, with numerical experiments on boundary layer and SPE10 problems demonstrating superior accuracy and stability on coarse meshes compared to Galerkin-based local solvers.

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