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EXACT LOCAL CONTROLLABILITY BY TRAJECTORIES FOR A QUASI-LINEAR PARABOLIC PDE WITH BILINEAR-CONTROL

CRISTIAN AMADOR LOLI PRUDENCIO
UNIVERSIDADE FEDERAL FLUMINENSE, IME, BRAZIL.
EMAIL:CRISTIANLOLI@ID.UFF.BR

Abstract. This paper deals with the exact control of a quasi-linear parabolic PDE with bilinear-control, in two and three dimension, that is to say $\Omega \subset \mathbb{R}^N, N = 2$ or 3

$$\begin{cases} u_t - \nabla.(a(u)\nabla u) + F(u) = h1_\omega u & \text{in } Q = \Omega \times (0,T) \\ u(x,t) = 0 & \text{in } \Sigma = \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{on } \Omega \end{cases}$$

The novelty in this work is the appearance of the spatial derivative of the solution instead of considering only the solution in the quasi-linear term (nonlinearity), here lies the difficulty of approaching said equation and in the second member the bilinear form. First, we study the linear problem through its adjoint problem and the associated Carleman inequality and then, we use the results obtained in the linear case to conclude the nonlinear problem by applying the Listernik´s Inverse Function Theorem (see for instance [1], [3], [4], [2]).

keywords: Parabolic quasi-linear problem · Exact controllability by trajectories· Carleman inequalities

References

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