A Bianchi-I model with non-commutativity

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Abstract

In the present work, a homogeneous and anisotropic Bianchi-I model is considered, in terms of configuration variables known as *Misner variables*. The matter content of the model is a radiation perfect fluid. First, we write the Hamiltonian of the geometric sector using the *ADM formalism* (Arnowitt-Deser-Misner). The Hamiltonian of the matter is obtained via the Schutz formalism, suitable for describing perfect fluids in Eulerian dynamics, providing a temporal parameter for the theory. The total Hamiltonian is considered in the cosmological gauge, where the lapse function is equal to one. Subsequently, non-commutativity is imposed on the model's canonical variables, which makes them obey new commutation relations via *deformed* Poisson brackets, now dependent on a non-commutative parameter γ, taken as small and constant. Furthermore, the non-commutative variables are written in terms of the usual commutative variables by means of transformations known as *Bopp shift*, which satisfy the deformed Poisson Brackets to first order in γ. Once this is done, we find the equations of motion for the non-commutative Hamiltonian, which leads to a system of differential equations for the scale factors, subject to certain constraints. The system is numerically solved and the quantitative behavior of the scale factors, one isotropic and two anisotropic, are studied. We do that, through individual variations of different parameters of the model, while keeping the other quantities fixed. In all cases the resulting model is expansive, with the isotropic factor growing forever, while the anisotropic factors become constants after some time. The presence of the γ term alters the dynamics of the commutative model and leaves residual contributions in the evolutions of the expansion factors. The quantitative behaviors of the expansion factors are also compared for different fluids and very similar results are found. The work shows that non-commutativity is compatible with the phenomenon of universe expansion for a Bianchi-I model.

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