

Convolution method

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Outline

- 1 Motivation
- 2 Convolution method
- 3 Examples
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Motivation

We are often interested in finding the probability distributions of functions of random variables.

In general, the Jacobian transformation method can be used to derive these probability distributions.



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Motivation

A particularly useful case arises when we consider a function of two independent random variables given by one of the four basic arithmetic operations: addition, subtraction, multiplication, or division.

Under these conditions, we can derive the probability distribution of the resulting random variable using convolution and related integral formulas.



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Convolution and related integral formulas

Suppose that X and Y are independent continuous random variables with probability density functions (PDFs) f_X and f_Y , respectively. Let

$$Z_1 = X + Y,$$

$$Z_2 = X - Y,$$

$$Z_3 = X \times Y,$$

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Probability density functions

The corresponding PDFs are given by

$$\begin{aligned}g_{Z_1}(z_1) &= \int_{-\infty}^{+\infty} f_X(t) f_Y(z_1 - t) dt = \int_{-\infty}^{+\infty} f_X(z_1 - t) f_Y(t) dt, \\g_{Z_2}(z_2) &= \int_{-\infty}^{+\infty} f_X(t) f_Y(t - z_2) dt = \int_{-\infty}^{+\infty} f_X(z_2 + t) f_Y(t) dt, \quad (1) \\g_{Z_3}(z_3) &= \int_{-\infty}^{+\infty} f_X(t) f_Y\left(\frac{z_3}{t}\right) \left|\frac{1}{t}\right| dt = \int_{-\infty}^{+\infty} f_X\left(\frac{z_3}{t}\right) f_Y(t) \left|\frac{1}{t}\right| dt, \\g_{Z_4}(z_4) &= \frac{1}{z_4^2} \int_{-\infty}^{+\infty} f_X(t) f_Y\left(\frac{t}{z_4}\right) |t| dt = \int_{-\infty}^{+\infty} f_X(z_4 \times t) f_Y(t) |t| dt, \\z_4 &\neq 0.\end{aligned}$$



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Examples

Example 1. Suppose we have an electrical circuit in which the current X and the resistance Y are both modeled as random variables. In particular, suppose that X and Y are independent continuous random variables with the following PDFs:

$$f_X(x) = 2x \mathbb{I}_{(0,1)}(x) \text{ and } f_Y(y) = \frac{y^2}{9} \mathbb{I}_{(0,3)}(y).$$

Using the convolution formula for the product of two independent random variables, determine the PDF of $Z = XY$.



Examples

Example 2. Suppose that $Z = X + Y$, where X and Y are independent standard normal random variables. Then,

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{x^2}{2}\right\} \mathbb{I}_{\mathbb{R}}(x) \text{ and } f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{y^2}{2}\right\} \mathbb{I}_{\mathbb{R}}(y).$$

Using the convolution formula for the sum of two independent random variables, determine the PDF of $Z = X + Y$.



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Bibliography

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Thank you!

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