Inference and Prediction in Inverse Gamma Regression: An Application to *Leptinaria unilamellata*

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Outline

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Motivation

Regression analysis is perhaps the most widely used statistical technique (Montgomery et al., 2021). In many applications, the response variable is continuous, strictly positive, and right-skewed.

For instance, one may consider the time until offspring release in individuals of the species *Leptinaria unilamellata* (d'Orbigny, 1835), particularly under unfavorable environmental conditions where retention within the oviduct may occur.

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Defined on the positive real line, the inverse gamma distribution has several advantages (Magalhães et al., 2021) and its reparameterized counter part (RIG; Bourguignon and Gallardo, 2020) enables a regression framework in which both the mean and dispersion parameters are modelled directly.

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The maximum likelihood (together with some of its variants) is the most widely used method of estimation (Lehmann and Casella, 1998).

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Reparameterized inverse gamma distribution

We say that a random variable Y follows a RIG distribution with mean $\mu > 0$ and precision parameter $\phi > 0$, if the distribution of Y admits the following probability density function (PDF):

$$f(y;\mu,\phi) = \frac{[\mu(1+\phi)]^{2+\phi}}{\Gamma(2+\phi)} y^{-(3+\phi)} \exp\left\{-\frac{\mu(1+\phi)}{y}\right\} \mathbb{I}_{\mathbb{R}_+}(y), \quad (1)$$

Inverse gamma distribution

where $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$ is the complete gamma function, and $\mathbb{I}_{\mathbb{R}_+}(y)$ is the indicator function that takes the value 1 if $y \in \mathbb{R}_+$ is true and 0 otherwise.

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RIG regression model

Let $Y_1, \ldots, Y_n$ be n independent random variables, where each Y_i, $i = 1, \ldots, n$, follows the PDF given in (1) with mean μ_i and precision parameter ϕ_i.

Suppose the mean and precision parameters of Y_i satisfy the following functional relations: $g_1(\mu_i) = \eta_{1i} = \boldsymbol{x}_i^\top \beta$ and $g_2(\phi_i) = \eta_{2i} = \boldsymbol{z}_i^\top \boldsymbol{\lambda}$,

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RIG regression model

where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_{p_1})^\top$ and $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_{p_2})^\top$ are vectors of unknown regression coefficients which are assumed to be functionally independent, $\boldsymbol{\beta} \in \mathbb{R}^{p_1}$ and $\boldsymbol{\lambda} \in \mathbb{R}^{p_2}$, with $p_1 + p_2 < n$, η_{1i} and η_{2i} are the linear predictors, and $\boldsymbol{x}_i = (x_{i1}, \ldots, x_{ip_1})^\top$ and $\boldsymbol{z}_i = (z_{i1}, \ldots, z_{ip_2})^\top$ are observations on p_1 and p_2 known regressors, for $i = 1, \ldots, n$.

RIG regression model

Furthermore, we assume that the covariate matrices $\boldsymbol{X} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)^\top$ and $\boldsymbol{Z} = (\boldsymbol{z}_1, \ldots, \boldsymbol{z}_n)^\top$ have rank p_1 and p_2, respectively.

The link functions $g_1 : \mathbb{R} \to \mathbb{R}^+$ and $g_2 : \mathbb{R} \to \mathbb{R}^+$ must be strictly monotone, positive, and at least twice differentiable such that $\mu_i = g_1^{-1}(\boldsymbol{x}_i^\top \beta)$ and $\phi_i = g_2^{-1}(\boldsymbol{z}_i^\top \boldsymbol{\lambda})$, with $g_1^{-1}(\cdot)$ and $g_2^{-1}(\cdot)$ being the inverse functions of $g_1(\cdot)$ and $g_2(\cdot)$, respectively.

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RIG regression model

For the log-likelihood function, the score vector and a detailed survey about maximum likelihood estimation, we suggest the original work of Bourguignon and Gallardo (2020).

Outline

Skewness coefficient

The Pearson's standardized third cumulant defined by $\gamma = \kappa_3/\kappa_2^{3/2}$, where κ_r is the rth cumulant of the distribution, is the most well-known measure of asymmetry.

When $\gamma > 0$ ($\gamma < 0$) the distribution is positively (negatively) skewed and will have a longer (shorter) right tail and a shorter (longer) left tail.

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Skewness coefficient

There is no closed-form for the skewness coefficient of the distribution of maximum likelihood estimators of the linear parameters in the RIG. The alternative is to derive an approximation up to order $n^{-1/2}$ from the general expression of Bowman and Shenton (1998).

Skewness coefficient

After a tedious algebra, we express the third cumulant of the respective set of MLE regressors parameters as,

$$\gamma_1(\widehat{\boldsymbol{\beta}}) = \kappa_3(\widehat{\boldsymbol{\beta}}) \odot \left\{\text{diag}\left(\widehat{\boldsymbol{K}}^{\boldsymbol{\beta},\boldsymbol{\beta}}\right)\right\}^{-3/2}, \tag{2}$$
$$\gamma_1(\widehat{\boldsymbol{\lambda}}) = \kappa_3(\widehat{\boldsymbol{\lambda}}) \odot \left\{\text{diag}\left(\widehat{\boldsymbol{K}}^{\boldsymbol{\lambda},\boldsymbol{\lambda}}\right)\right\}^{-3/2},$$

Skewness coefficient

where

$$
\begin{aligned}
\kappa_3(\widehat{\boldsymbol{\beta}}) &= \boldsymbol{Q}^{(3)}_{\beta\beta}\boldsymbol{M}_1\mathbf{1} + \left(\boldsymbol{Q}^{(2)}_{\beta\beta} \odot \boldsymbol{Q}_{\beta\lambda}\right)(\boldsymbol{M}_2 + 2\boldsymbol{M}_3)\,\mathbf{1} \\
&\quad + \left(\boldsymbol{Q}_{\beta\beta} \odot \boldsymbol{Q}^{(2)}_{\beta\lambda}\right)(2\boldsymbol{M}_4 + \boldsymbol{M}_5)\,\mathbf{1} + \boldsymbol{Q}^{(3)}_{\beta\lambda}\boldsymbol{M}_6\mathbf{1}, \qquad (3) \\
\kappa_3(\widehat{\boldsymbol{\lambda}}) &= \boldsymbol{Q}^{(3)}_{\lambda\beta}\boldsymbol{M}_1\mathbf{1} + \left(\boldsymbol{Q}^{(2)}_{\lambda\beta} \odot \boldsymbol{Q}_{\lambda\lambda}\right)(\boldsymbol{M}_2 + 2\boldsymbol{M}_3)\,\mathbf{1} \\
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\end{aligned}
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The remaining matrices a presented in Magalhães et al. (2025).

Prediction interval

Prediction intervals are constructed for future realizations of a random variable, whereas confidence intervals are constructed for unknown but fixed model parameters.

Suppose that a prediction for the response of a new observation, denoted by Y_0, is required. It is natural to assume that $Y_0 \sim \mathrm{RIG}\Big(\exp\{\boldsymbol{x}_0^\top \beta\}, \exp\{\boldsymbol{z}_0^\top \boldsymbol{\lambda}\}\Big)$, independently of $\boldsymbol{Y}$.

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Prediction interval

After some algebra involving asymptotic theory, omitted here for the sake of brevity, an asymptotic prediction interval for Y_0 is given by,

$$\mathrm{PI}\left[Y_0,\, 1-\alpha\right] = \left[\frac{\exp\{\boldsymbol{x}_0^\top \widehat{\boldsymbol{\beta}}\}\left(1+\exp\{\boldsymbol{z}_0^\top \widehat{\boldsymbol{\lambda}}\}\right)}{\mathcal{G}_{1-\alpha/2}(\exp\{\boldsymbol{z}_0^\top \widehat{\boldsymbol{\lambda}}\}+2,1)}; \frac{\exp\{\boldsymbol{x}_0^\top \widehat{\boldsymbol{\beta}}\}\left(1+\exp\{\boldsymbol{z}_0^\top \widehat{\boldsymbol{\lambda}}\}\right)}{\mathcal{G}_{\alpha/2}(\exp\{\boldsymbol{z}_0^\top \widehat{\boldsymbol{\lambda}}\}+2,1)}\right], \quad (4)$$

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Simulation study

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Table 1: Sample and $n^{-1/2}$ skewness coefficients.

$\boldsymbol{\beta}$	$\boldsymbol{\lambda}$	Estimator	$n = 20$		$n = 30$		$n = 40$	
			$\lvert\rho\rvert$	$\lvert\gamma_1\rvert$	$\lvert\rho\rvert$	$\lvert\gamma_1\rvert$	$\lvert\rho\rvert$	$\lvert\gamma_1\rvert$
(4.50, -0.25, -1.00)	(3.0, -0.1, 0.5)	β_0	0.304	0.625	0.310	0.428	0.203	0.360
		β_1	0.068	0.177	0.029	0.024	0.003	0.034
		β_2	0.071	1.769	0.104	1.482	0.155	1.037
		λ_0	0.854	0.416	0.786	0.241	0.924	0.191
		λ_1	0.011	0.326	2.249	0.237	2.775	0.214
		λ_2	0.481	1.331	0.027	1.066	0.509	0.787

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Table 2: Comparison for PI.

Case	Set	Scenario	n=20			n=30			n=40			n=60			n=100		
			90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%	90%	95%	99%
A	1	I	0.768	0.830	0.909	0.831	0.892	0.957	0.858	0.916	0.971	0.869	0.924	0.977	0.881	0.934	0.983
		II	0.836	0.895	0.957	0.862	0.919	0.976	0.872	0.927	0.977	0.875	0.934	0.987	0.893	0.941	0.987
		III	0.855	0.913	0.969	0.866	0.923	0.977	0.875	0.932	0.980	0.890	0.942	0.987	0.888	0.940	0.984
	2	I	0.762	0.823	0.903	0.830	0.888	0.953	0.852	0.913	0.971	0.864	0.922	0.975	0.882	0.933	0.983
		II	0.834	0.899	0.962	0.861	0.920	0.976	0.877	0.932	0.982	0.880	0.937	0.986	0.885	0.942	0.987
		III	0.855	0.914	0.970	0.869	0.924	0.976	0.877	0.932	0.981	0.886	0.940	0.985	0.891	0.943	0.987
B	1	I	0.794	0.858	0.934	0.841	0.901	0.960	0.853	0.912	0.972	0.874	0.929	0.980	0.883	0.940	0.986
		II	0.838	0.897	0.962	0.860	0.917	0.978	0.868	0.924	0.976	0.882	0.938	0.986	0.886	0.938	0.985
		III	0.845	0.909	0.969	0.864	0.923	0.979	0.877	0.933	0.982	0.881	0.938	0.984	0.888	0.944	0.988
	2	I	0.806	0.870	0.938	0.844	0.906	0.965	0.855	0.909	0.969	0.873	0.931	0.979	0.890	0.942	0.987
		II	0.838	0.900	0.968	0.865	0.919	0.976	0.872	0.928	0.980	0.880	0.934	0.984	0.891	0.941	0.988
		III	0.849	0.910	0.971	0.863	0.923	0.982	0.877	0.931	0.984	0.881	0.936	0.984	0.891	0.942	0.989

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Leptinaria unilamellata dataset

From Daniel (2013), we illustrate the proposed method by analyzing a data set in which the response variable is the intraoviductal retention duration in individuals of the species *Leptinaria unilamellata* (d'Orbigny, 1835, y),

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with covariates representing environmental conditions that may affect the duration of intraoviductal retention (x_2) and the number of offspring released (x_3) by each individual was recorded by direct visual counting (Arct et al., 2025; Seki et al., 2010; Setiaji et al., 2021).

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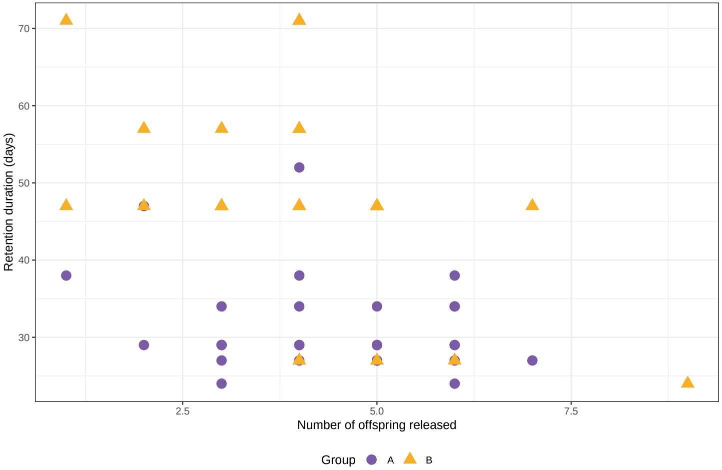

Figure 1: Scatterplot.

Leptinaria unilamellata dataset

We assumed that $y_i \sim \text{RIG}(\mu_i, \phi_i)$ with

$$\log \mu_i = \beta_1 + \beta_2 \times \text{Group:B}_i + \beta_3 \times \text{Off}_i$$

$$\log \phi_i = \lambda_1 + \lambda_2 \times \text{Group:B}_i,$$

$i = 1, \ldots 52.$

Table 3: Estimates for the Leptinaria unilamellata dataset.

Parameter	Estimate	SE	z-value	p-value	γ_1
β_1	3.683	0.084	43.683	< 0.001	0.146
β_2	0.410	0.063	6.501	< 0.001	0.381
β_3	-0.055	0.018	-3.094	0.003	0.000
λ_1	3.482	0.269	12.928	< 0.001	0.104
λ_2	-0.932	0.414	-2.251	0.029	-0.047

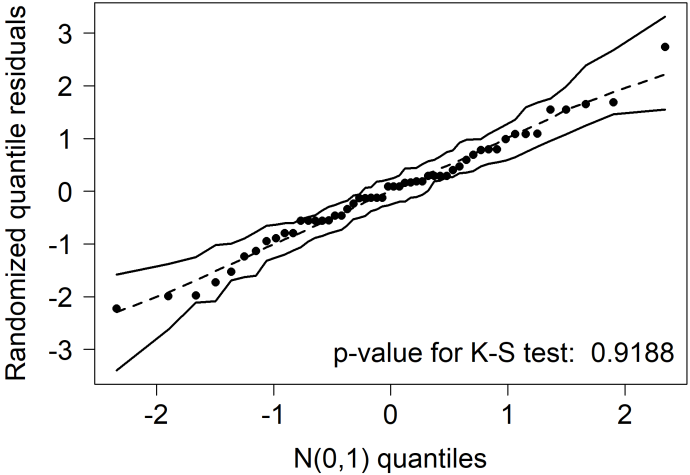

Figure 2: QQ plot for quantile residuals.

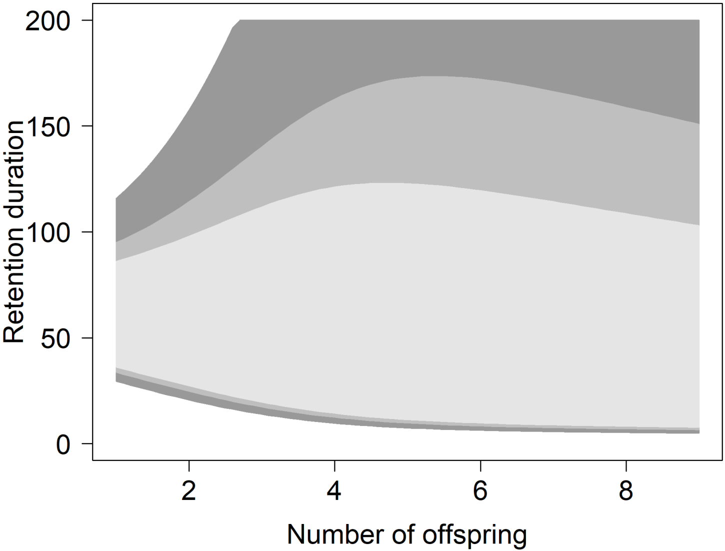

Figure 3: 90%, 95% and 99% PI of the retention duration.

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Concluding remarks

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1. FAPEMIG/Brazil (grants APQ-03202-26 & RED-00133-21);
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Concluding remarks

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Thank you!

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