# DISAGREEMENT, SUPERIOR PREDICTING ABILITY, AND PERSISTENCE: THE EXPECTATIONS OF BRAZILIAN INFLATION

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# DISAGREEMENT, SUPERIOR PREDICTING ABILITY, AND PERSISTENCE: THE EXPECTATIONS OF BRAZILIAN INFLATION[1]

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## Abstract

The present work aimed to study the divergence problem, starting from the statistical verification of the superior predicting ability of institutions that take part in the research (Focus-Market Report) of the Gerin/Bacen. For such, it was applied the multiple comparison of de predictions, according to Hansen (2005). The disagreement was confirmed by empirical results indicating that it increases (reduces) with increasing (reducing) inflation. The application of the SPA test, in short and long term analyses, suggested strength in the disagreement in the expectations of inflation, in which some models/institutions exert superior predicting ability. It is also possible to observe the presence of some degree of inertia in the dynamics of the expectations, in which the measures of persistence suggest sharp shock effects that diminish as time passes, in most of the cases analyzed. We concluded that disagreement of inflation are caused by superior predict ability with some degree of persistence inflation.

*Keywords*: Inflation; Expectations; Monetary policy.
*JEL classification*: D84; E31; E42; E47; E52.

## Resumo

O presente trabalho buscou estudar a divergência a partir da verificação estatística da habilidade preditiva superior de instituições que participaram da pesquisa "Focus-Relatório de Mercado" da Gerin/Bacen. Para tanto, aplicou-se comparação múltipla de previsões, conforme Hansen (2005). Os resultados empíricos indicaram que o desacordo nas expectativas existe e que ele se eleva (reduz) com o aumento (queda) da inflação efetiva, e que, ainda, por algumas vezes, tende a aumentar quando a inflação muda bruscamente. Já a aplicação do teste SPA, em análises de curto e longo prazos, implicou robustez quanto ao desacordo nas expectativas de inflação, em que alguns modelos/instituições exercem habilidade preditiva superior. Pode-se observar também a presença de algum grau de inércia na dinâmica das expectativas, em que as medidas de persistências sugerem efeitos acentuados do choque ocorrido, mas que tais efeitos decaem com o decorrer do tempo na maioria dos casos analisados.

*Palavras-chave*: Inflação; Expectativas; Política Monetária.
*Classificação JEL*: D84; E31; E42; E47; E52.

## 1. INTRODUCTION

In Brazil, after some years of high economic growth, the 80s were marked by the conjunction of two factors: increase in the inflation rate and decrease in the growth rate. This situation continued until the beginning of the 90s, leading the country to adopt seven stabilization plans in less than ten years. Nowadays, there is a certain consensus that price stability brings several advantages for self-sustained growth.

For such, it is necessary to convince society that, in medium and long terms, higher inflation does not generate more growth. On the contrary, it creates an unfavorable environment for investments and penalizes the poorest population, promoting income concentration.

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To avoid losing the price stability reached with the Plano Real in 1994, it was implemented the current inflation targeting regime. In this regime, the Central Bank of Brazil (Bacen) committed itself to act in a way to guarantee that the inflation observed be in line with a pre-established goal, guided by some basic principles implemented in the realm of the  Monetary Policy Committee (Copom). Among these principles, it is pointed out the evaluation of inflation future trend.

Besides, the Bacen and a greater number of central banks in the world (New Zealand, USA, Canada, Chile, United Kingdom, Sweden, Peru, Mexico, Brazil and Israel, among others) have dedicated special attention to the regular monitoring of the private sector expectations. With that, the monetary authorities have a crucial instrument of monetary policy management, since the expectations play a relevant role in the inflationary dynamics itself. It is worth to add that the main central banks of the world adopt price stability as the main objective of the monetary policy, either explicitly or implicitly, as is the case of the United States[4] (BANK OF ENGLAND, 1999; SVENSSON, 1997; WOODFORD, 2003).

Expectations are important because they are a component in the formation of prices, wages and economy contracts, besides determining the term structure of *ex-ante* real interest rates associated to a determined nominal interest curve, which, by its turn, affects the private decisions of consumption and investments.

The monitoring of expectations, especially inflation expectations, is even more important in countries which follow the inflation targeting regimes. According to this regime, when conducting the monetary policy, the Central Bank searches to coordinate the formation of the private expectations.

In Brazil, the Bacen weekly issues the FOCUS- Market Report by Executive Management of Relationship with Investors (Gerin). This document brings market projections and the evolution of important macroeconomic variables, besides pointing out the expectations of the institutions with the greatest predicting ability, the so-called Top 5 institutions[5].

The Gerin, created in April 1999, is also part of the structure of the monetary regime of inflation targeting, aiming to improve the communication between the Bacen and the private sector, focusing on domestic and foreign investors[6]. Then, in May 1999, the Research on Market Expectations began. Although it was created in May 1999, only with the increasing relevance of the expectations monitoring for the monetary policy, in November 2001, the *internet homepage* of the Market Expectation System was created, with the objective of speeding the collection process and minimizing the occurrence of misinformation (MARQUES, FACHADA and CAVALCANTI, 2003, page 10).

Currently, the research monitors the market expectations for different prices indexes, the Gross Domestic Product growth and industrial production, exchange rate, Selic interest rate, tax variables and the indicators of the external sector.

However, a recurrent criticism to the Gerin expectation research refers to the fact that it is based in a sample basically composed of financial sector specialists. Therefore, it would not be a *proxy* of the economic agents expectations, but instead, of a small part out of the whole.

In fact, according to the BACEN (2004, p. 116), out of 104 informants, 84 were financial institutions (52 commercial banks, 23 investment funds, and 9 brokerage firms  and  securities), 14 consultancy companies and 6 non-financial companies and class entities. The Bacen argues that, in practice, companies and entities, regardless of the sector to which they belong, present smaller capacity to respond to macroeconomic changes than the participants of the financial sector and

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[4] Mankiw (2006).

[5] More information about the Central Bank System of Market Expectations can be found in Marques, Fachada and Cavalcanti (2003).

[6] The Gerin manages a list of thousands of e-mails, to which it sends the main records of the Bacen. Besides, it produces its own reports, all available at www.bcb.gov.br, including: Focus Market Report; Time Series of the Market Expectations: weekly updated spreadsheets, containing the daily data of the market expectations for the main economy variables since 2001; Top 5; "Frequently Asked Questions" Series; Weekly Review of Open Market.

consultancy companies, due to the very nature of their professional activity, mainly in the presence of data of high frequency and/or in an environment of volatilities[7].

If some economic agents linked to companies and entities do not respond in the same speed as the financial analysts do, then it is reasonable to imagine the possibility of the existence of divergences in expectations as to the macroeconomic variables; in the present case, the economic prices.

Some works in Brazil had the only objective of verifying whether the Bacen prediction is similar to that of the market, (BACEN, 2003), and if time series models perform a better prediction than the Bacen does (LIMA and CÉSPEDES, 2003). The results of these authors showed that some simple autoregressive models are able to generate predictions more exact than those from the Bacen, and that the market expectations are inflation predictions statistically similar to those of the Bacen. So, it is undeniable that the comparison between the estimates or expectations and the results would be an important determinant of the prediction capacity of the models considered, even if their structures are not specified.

So, with the instruments used in the financial area, discussed in section 2, for the comparison of the predicting capacity of the models, it would be possible to verify important results in the supposition that some model would present superior predictive ability to the others. Among them:

(i) even with some consideration among the sector expectations, the superior predictive ability would restrain the capacity of the Bacen even in the specification of the Selic interest rate-target due to the expectation divergence as Mankiw, Reis and Wolfers (2003);

(ii) distortions caused by the phases of the economic cycles and the inflation path, either ascending or descending , would be observed; and

(iii) the consequences of prediction errors about variables, initially mentioned by Lima and Céspedes (2003), would be unknown, mainly as to the long term horizon.

In this case, it is important to verify statistically the superior predictive ability of the institutions that take part of the "Focus-Market Report", spreading their expectations concerning the Brazilian official inflation, measured by variation of the Broad Consumer Price Index (IPCA), as well as the alternatives available to the economic agents. For such, it is used the new *Superior Predictive Ability* (SPA) *test* of Hansen (2005), in which "models" are proposed as *benchmark,* for both short and long terms.

With the *random walk* model, representing the *benchmark*, it was investigated the inferiority of the prediction of the institutions selected in the presence of a singular model, as well as the problem of the disagreement in the inflation expectations, in case some agents adopt this model. Like the others, it is verified if it is reasonable to consider, for example, that the Bacen Focus/Top5 daily surveys Bacen are good forecasters of the inflation rate in short and long terms and if they corroborate results in literature.

In part, the contribution of this article follows the reasoning of evaluating the superior predictive ability of the institutions in face of simple *benchmark* models, such as *random walk* and AR(p), but it is distinguished because it is a statistical evaluation of the problem and it contributes to the current discussion on the disagreement in the inflationary expectations, something that is not performed by Lima and Céspedes (2003). It is also proposed a new TOP 5 (TOP 5*), comparing it with the other models, including the Gerin/Bacen TOP 5, also, in other part, evaluating the inertia degree in the dynamics of the inflation expectations from the ratio of variances, systematically used to measure shock persistence in the long run.

The paper unfolds as follow: Section 2 discusses theoretical foundations on the relevance of the expectations in economy; Section 3 presents the models of superior predictive ability in *snooping data*, the new test of superior predictive ability proposed by Hansen (2005), and Persistence Measures; Section 4 shows the empiric results achieved, their interpretations and a

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[7] Companies and organizations, independently of the sector to which they belong, must meet certain criteria of admission and regularity so that they can take part in the research on market expectations.

discussion of the disagreement problem in expectations; and, finally, Section 5 closes with the conclusions.

## 2. THEORETICAL FOUNDATIONS

For almost half a century, inflation expectations were considered a key-variable in the specification of macroeconomic models. Started by Friedman (1968) and Phelps (1969), the expectations would explain the phenomenon of the stagflation. Years later, Lucas (1973) gave more attention to this variable by considering the rational expectations in an expectations-augmented Phillips curve. Such models would solve the problem of the perfect prediction in the models of the neoclassical intertemporal equilibrium.

An important postulate on the role of the expectation divergence has been considered in the economic literature since the contribution of Lucas (1973). In the 80s, based on the work of Taylor (1980), Roberts (1995) suggested a New Keynesian Phillips Curve as a model, with the dominance of price rigidity, in which the current inflation would depend on the product and the expectations of the future inflation. However, in such model, some puzzles were detected, namely: in disinflation, there would be expansion instead of recession; the persistence of inflation would not be explained; and, finally, the gradual response to real variables shocks would not be supported by this model

Recently, however, Mankiw and Reis (2002) structured a model of the New Keynesian Phillips Curve, in which the rigidity would be of information, instead of prices, causing a slow adjustment in prices.

Based on the informational rigidity, Mankiw, Reis and Wolfers (2003) suggested that the inflation expectation disagreement would be a key-variable to be considered in the macroeconomic dynamics and, for such, they considered three different sources of expectation statistics, which are: *Michigan Survey Consumer Attitudes and Behavior, Livingston Survey* and *Survey of Professional Forecasters*. Based on the analysis of the information, the authors concluded that the divergence would enlarge with the increase of the inflation and that this divergence would tend to augment with a sudden inflation change, regardless of the movement direction. Still, the expectation dispersion of the intra-group inflation would enlarge the variance; and finally, that there would not be any clear relation between the divergence and real economic variables. In other words, none of the models clarifies or structures the expectation divergence as a macroeconomic variable.

So, the debate was open around the predictive capacity of the models, without a specific goal. Important results have been observed in literature. For Branch (2004), there are many inflation predicting models, and the traditional approach of limited rationality considers that the agents would use an adaptive rule of their predictions. In other words, the predicting ability of a specific model upon the set of alternative models would make it possible to corroborate the hypotheses of divergence and expectational heterogeneity, given different costs of achievement of relevant information on the prediction of such models, since the agents or groups do not have any econometrists on duty. Besides, in the ARED (Adaptively Rational Equilibrium Dynamics) models, the agents would choose a prediction function among a set of alternatives, with an increasing cost of ordination due to the sophistication of the models. Then, the probability of a predictor selection would depend on the net benefits in comparison to the respective models.

Branch (2004) widened the analysis of such models when he considered inflation adaptive expectations 12 steps ahead, in which the optimality of a predictor would depend on the agent that chooses it, thus suggesting a heterogeneous rationality in the selection process and, with that, an expectational divergence.

According to Döpke, Dovern, Fritsche and Slacacek (2006), the informational rigidity would be observed for countries like Germany, France, Italy and the United Kingdom, so that, for Italy, the readjustment of the expectations would occur in only six months and, for the other countries, the gap would be of one year, thus characterizing the informational rigidity. So, the superiority and the consistency of the informational rigidity model would have solved the puzzles around the inflation persistence and the reductions costs.

However, the comparison of the predictive power of the price rigidity models, informational rigidity and price rigidity with an indexation mechanism, implemented by Korenok (2008) and Korenok and Swanson (2007), restricts the results of Döpke, Dovern, Fritsche and Slacacek (2006), because the dominance would be of the price rigidity models, in periods of increasing inflation.

Due to such possibility, the consideration of the comparison of the inflation prediction models is very important, due to the expectational divergence phenomena, the predictive ability, and the problem of inflation persistence as well. In the scope of the Corporative Finances, the empiric literature has presented relevant methods and results likely to be used in the Macroeconomics field.

Using the instrument started by Lo and Mackinlay (1990), who sought to quantify the bias of the structures of the *Capital Asset Price Model* (CAPM) prediction models and the *Asset Price Theory* (APT) model for the *snooping data* of the North American financial market, Hansen and Lunde (2005) confronted 330 models of the ARCH structure for the exchange rate German Mark/Dollar and the return of the actions of the IBM and established the GARCH (1, 1) model as *benchmark*. After carrying out the superior predictive ability test of Hansen (2005) and the White (2000) test, they concluded that the GARCH (1, 1) model is superior, at least for the exchange rate.

Koopman, Jungbacker and Hol (2005) compared the models of unobserved components (UC), ARFIMA long memory models, stochastic volatility models (SV) and autoregressive conditional heteroskedasticity models (GARCH) and, based on the daily data of approximately 7 years for the index of Standard & Poor's 100 (S&P 100), the long memory models were considered superior to the others.

In Brazil, using the superior predictive ability test of White (2000), the *Reality Check for Data Snooping*, Saffi (2003) demonstrated that none of the 14630 strategies of technical analysis would be able to generate statistically significant returns because of the consideration of the *snooping* data effects. So, the weak-form market efficiency hypothesis would be proven. Concerns with the Bacen predicting capacity was suggested by Lima and Céspedes (2003) when the authors compared the following prediction models: AR(2), AR(4), ARIMA (0,1,2) and ARIMA (2,1,4) to the FOCUS and TOP 5 predictions and concluded that the accuracy of the Bacen as to long horizons of prediction would be questioned. Although the authors question the authority capacity, especially concerning long term horizons, since they would be considered in the specification of the alterations in the Selic interest rate-target, the authors do not consider the structural diversity of the FOCUS results. They neither present any test with enough statistical rigor so that a superior prediction model could be presented. Finally, they do not evidence which would be the effect of the authority's limited capacity on relevant economic variables.

With such objectives, the present work aimed to adopt the Hansen procedure for the SPA test to present, based on the results, some evidence about the divergence, capacity and even a discussion on the effects of these errors.

## 3. SUPERIOR PREDICTIVE ABILITY MODELS IN SNOOP DATA

The origin of the concern about the use of the same financial dataset for the generation of prediction models, although distinguished in their estimates, reminds the work of Cowles (1933), entitled "Can Stock Market Forecasters Forecast?"

This author analyzed the proposals spread by 45 brokerages from January 1928 to July 1932, during the Great Depression period. His motivation was to demonstrate that the same dataset on prices and variations of the assets would generate different proposals for clients of those agencies. Then, the genesis of superior predictive ability issue would be the joint treatment of prediction models, the use of enclosing prediction models, in order to test efficiency, as well as equal predictive ability (West, 1996).

### 3.1. The bias problem in *Snooping Data*

A treatment based on a similar idea was suggested by White (2000) and defined as bias in *snooping data*. The occurrence of *snooping data*, according to this author, would take place when a

set of data was used more than once with the objective of inferring or selecting a model among a set of alternatives. Such practice would be dangerous for researchers and for market analysts, if careful comparisons were not performed.

In the scope of the equal predictive ability test, there would be concerns on the binary comparison between the *benchmark* model, considered the most efficient, and an alternative model, represented by a null (simple) hypothesis of equal predictive ability. In the superior predictive ability (SPA) test, there would be a multiple comparison between the alternative models, composed in a nullity hypothesis that represents inferiority to the *benchmark* model, and the alternative hypothesis, superior to the *benchmark*.

For the case of the SPA test, the following elements would be necessary: alternative models different from the *benchmark*; a decision rule based on composed hypotheses; and restrictions on the distribution, according to the fundamental asymptotic properties for the specification of optimal tests, with the possibility of distributional variations.

White (2000) specified a test for the superior predictive ability that would be more powerful and less sensitive to poor and irrelevant alternatives. In other words, additional hypothetical information would have little effect on the predictive capacity of the structure considered.

In the structure *Reality Check*, White (2000), *l* alternative predictions would be compared to a basic model, and the predictive ability of each one of the alternatives would be considered from an expected loss function, as specified by the equation 1:

$$\hat{f}_{k,t+1} = logL_K(Y_{t+1}, X_{k,t+1}, \hat{\beta}_{k,t}) - logL_0(Y_{t+1}, X_{0,t+1}, \hat{\beta}_{0,t}), k = 1, ... l \qquad (1)$$

where:
$L_k(\cdot) = $ log-predictive likelihood of the $k$ $(k=1,....,l)$ model, alternatives to the *benchmark*.

The test, therefore, would impose the following joint decision rule between the alternatives to the *benchmark* model and the respective model, namely:

$$H_0: max_{k=1,...,l}E(f_k^*) \leq 0 \qquad (2)$$

In which the hypothesis of nullity establishes that the alternatives would not be superior to the *benchmark* model.

With this framework, the basic alternative would be a limit test like Bonferroni[8], which could be specified as:

$$Prob\left\{ \bigcup_{i=1}^{l} \left( P_i \leq \frac{\alpha}{l} \right) \right\} \leq \alpha, (0 \leq \alpha \leq 1) \qquad (3)$$

The equation (3) would represent the *inequality of Bonferroni*, built from the n-statistics of the p-values corresponding to $P_1, ...., P_l$, which would represent the composition $H_1, ..., H_l$, for which $H_0 = \{ H_1, ..., H_l \}$ would be rejected if some of the p-values were lower than $\alpha/l$, in which $\alpha$ represents the probability of the occurrence of the Type I Error to reject the nullity hypothesis when it is true, in independent tests. In general, the inequality of *Bonferroni* must be used in multiple tests, to determine a superior limit for the $\alpha$ significance level, probability of making the Type I Error.

In spite of the simplicity of the test, due to its tractability, it would be limited in power if highly correlated individual tests were considered, which seems to be the case of the data on predictions of some macroeconomic variables like inflation, exchange rate, growth, among others. If very closed models were considered in joint test, in other words.

So, White (2000) was concerned about the settle of an optimal test based in the restrict maximization in which a smaller Type II Error, could accept $H_0$ when it is false, given the test

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[8] Consult Simes (1986) for a modified version of the test in the ordering of the probabilistic values.

power ($1 - \beta$). In the case of an unbiased test, according to Spanos (1989), in which $H_0$ would be really true, its power should be equivalent to its size. Therefore, Sullivan, Timmerman and White (2003) proposed two alternatives of simulation for the achievement of parameters for the test: Monte Carlo simulation and *bootstrap* procedure, while White (2000) proved that the distribution should be approximately asymptotic normal.

### 3.2. Superior predictive ability (SPA) – Hansen's proposal

Hansen (2005) established an alternative for White (2000), employing a different statistical test, based on a dependent sample distribution, under a null hypothesis. Then, the contributions of Hansen (2005) would be: a theoretical analysis of the test problem, exposing some of their important aspects, in which the inclusion of poor and irrelevant alternatives could be manipulated, thus altering the results, for the benefit of one of the models; an detailed explanation of the bootstrapping procedure implementation; and, an application for the North-American inflation, using the *random walk* model *as benchmark*.

The SPA test, proposed by Hansen (2005), is a test that can be used to compare the performances of two or more forecasting models. The forecasts are evaluated using a pre-specified loss function, and the "best" model forecast indicates the smaller expected loss. Two loss functions were pre-suggested by the author: "the mean squared error (mse) and mean absolute deviation (mad)".

In analytical terms, the application of the SPA test, Hansen and Lunde (2007), would start with the specification of an expected loss function.

Let $L\left(Y_t, \hat{Y}_t\right)$ denote the loss under the supposition that the institution/model has made the prediction, $\hat{Y}_t$, when, in fact, the realized value was $Y_t$; then the performance of model $k$, relative to the *benchmark* model (at time $t$), can be defined as:

$$X_k(t) = L\left(Y_t, \hat{Y}_{0t}\right) - L\left(Y_t, \hat{Y}_{kt}\right), k = 1, \dots, l; t = 1, \dots, n. \tag{4}$$

The issue of interest is to verify if any of the models are better than the *benchmark* model. For such, it is tested the hypothesis that the *benchmark* model is the best forecasting model among those analyzed in the test. This hypothesis can be expressed parametrically as follows:

$$H_0: \mu_k = E[X_k(t)] \leq 0, \forall\, k = 1, \dots, l. \tag{5}$$

For notational convenience, the *l*-dimension vector µ is defined by

$$\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_l \end{pmatrix} = E\begin{pmatrix} X_1(t) \\ \vdots \\ X_l(t) \end{pmatrix} \tag{6}$$

Provided that a positive value of $\mu_k$, corresponding to the model $k$, be better than the *benchmark*, it is possible to test the hypothesis $H_0: \mu_k \leq 0$, for $k = 1, \dots, l$. The equivalent vector formula would be $H_0: \mu \leq 0$.

A way of testing this hypothesis is to consider the test statistic[9], defined by:

$$T_n^{SM} = \max_k \frac{n^{\frac{1}{2}} \bar{X}_k}{\hat{\sigma}_k} \tag{7}$$

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[9] A consistent estimator of the asymptotic variance, $\hat{\sigma}_k^2 = \lim_{n \to \infty} var\left(n^{\frac{1}{2}} \bar{X}_k\right)$

A consistent estimator to $\hat{\sigma}_k^2$ is calculated using the *bootstrap* method. The term "*SM*" refers to the *standardized maximum*. Under the regularity conditions it holds that:

$$n^{-\frac{1}{2}}T_n^{SM} = \max_k \frac{\overline{X}_k}{\hat{\sigma}_k} \xrightarrow{p} \max_k \frac{\mu_k}{\sigma_k} \tag{8}$$

which will be greater than zero if and only if $\mu_k > 0$, for some $k$. Thus, it is possible to test $H_0$ using the test statistic $T_n^{SM}$. The problem is to derive the distribution of $T_n^{SM}$, under the presupposition of a true null hypothesis. To test multiple inequalities is more complicated than to test equalities (or a single inequality) because the distribution is not unique under the null hypothesis. However, a consistent estimate for the *p*-value can be achieved by the SPA program of Hansen (2005), as well as an upper and a lower bound.

So, the structures of the hypothesis to be tested by the SPA model would be based on the statistics of the expressions (5) and (6), respectively:

$$T_n^{RC} = \max_k \sqrt{n}\, \overline{X}_k \tag{9}$$

$$T_n^{SPA} = \max_k \frac{\sqrt{n}\, \overline{X}_k}{\hat{\sigma}_k} \tag{10}$$

The difference among the statistics (9), for the *Reality Check*, White (2000), and (10), for the SPA case, Hansen (2005), is that the statistics of the SPA would be less sensitive to the inclusion of the poor and irrelevant alternatives. Besides, the model is more powerful because it reduces the probability of the occurrence of the Type II Error. Even, the variance would be asymptotically consistent, obeying the conditions of regularity, as proposed by Sullivan, Timmerman, and White (2003).

### 3.3 Persistence Measures

One important persistence measure, specifically, was achieved from the alterations in test statistics proposed by Wright (2000), and it is more adequate in the presence of discordant observations, either *outliers* or *inliers*.

Inertia is a persistence of long duration. In time series analysis, the completely inertial case corresponds to the *random walk* process, in which a shock of one per cent becomes completely persistent.

Be $P_t$ a random time series with $T+1$ observations $(P_0, P_1, ..., P_T)$, then $\pi_t = \Delta P_t$ implies $T$ observations. The ratio of the variance indexed by the k index is defined by:

$$V_k = \frac{1}{k}\frac{Var(P_{t+k}-P_t)}{Var(P_{t+1}-P_t)} \tag{11}$$

It is possible to estimate $V_k$ replacing $Var(P_{t+k} - P_t)$ and $Var(P_{t+1} - P_t)$ by its correspondent estimators, achieving:

$$\hat{V}_k = \left\{ \frac{1}{(T-k+1)k}\sum_{t=k}^{T}(\pi_t, \cdots, \pi_{t-k+1} - k\hat{\mu})^2 \right\} \Big/ \left\{ \frac{1}{T}\sum_{t=1}^{T}(\pi_t - \hat{\mu})^2 \right\} \tag{12}$$

where: $\hat{\mu} = T^{-1}\sum_{t=1}^{T}\pi_t$. This estimator will be denoted by $\hat{V}_{2k}$.

If $P_t$ follows a *random walk*, the ratio above is approximately similar to one. On the other hand, if the $P_t$ series is stationary, the ratio in (12) gets close to zero when $k$ increases. Cochrane (1988), studying the asymptotic properties of $V_k$, provided an approximation for the expression (11):

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$$V_k \cong 1 + 2\sum_{j=1}^{k}\left(1 - \frac{j}{k}\right)\rho_j \tag{13}$$

in which $\rho_j$ is the *j-th* autocorrelation of $P_t$. The expected value of $V_k$, under the null hypothesis of serial independence, is achieved observing that the *j-th* sample autocorrelation has an expected value *[-1/(T - j)]*, as it is demonstrated in Kendall and Stuart (1976), and thus:

$$E(V_k) = \frac{2-k}{k} + \frac{2}{k}\sum_{j=1}^{k-1}\frac{T-k}{T-j} \tag{14}$$

It is possible to estimate $V_k$ by replacing the population autocorrelations $\rho_j$ in (13) by the corresponding sample autocorrelations $\hat{\rho}_j$, thus achieving:

$$\hat{V}_k \cong 1 + 2\sum_{j=1}^{k}\left(1 - \frac{j}{k}\right)\hat{\rho}_j \tag{15}$$

This estimator will be denoted by $\hat{V}_{1k}$. It is achieved from (15) which $\hat{V}_{1k}$ is a linear combination of the first *k* sample autocorrelations of the first difference of the series with weights arithmetically decreasing. For its construction, $\hat{V}_{1k}$ is superiorly non-negative and non-limited and its distribution is asymmetric in finite samples. There is a severe bias in $\hat{V}_{1k}$. For example, for the *random walk* the average of $\hat{V}_{1k}$ is approximately *[(T - k + 1)/T]*, instead of the expected value 1. That is why it is common to multiply the ratio of the estimated variances by *[T/(T -k+1)]* to correct this negative bias.

It is important to observe that the use of $\hat{V}_{1k}$ has many advantages. Particularly, it presents an asymptotic standard deviation smaller than other more simple estimators of $V_k$, for example, some estimator that attributes equivalent weights for all the *k* sample autocorrelations.

A spectral analysis result (PRIESTLEY, 1982) provides the asymptotic standard error of $\hat{V}_{1k}$ that can also be achieved from the formula of Bartlett (1946):

$$s.e.\left(\hat{V}_{1k}\right) = \frac{\hat{V}_{1k}}{\sqrt{\frac{3T}{4k}}} \tag{16}$$

Using signals of series, it is possible to construct two other measures of persistence. For some series, $\pi_t$ defines the function $f(\pi_t, q) = \frac{1}{2}$ if $\pi_t \geq q$ and $f(\pi_t, q) = -\frac{1}{2}$, in opposite case.

Given $s_{1t} = 2f(\pi_t, 0)$, it can be seen that $s_{1t}$ is independent and identically distributed with average 0 and variance 1. Each one is equivalent to 1 with probability of 1/2 and it is equivalent to -1 with probability of 1/2. $s_{2t} = 2f(\pi_t, \hat{\mu})$ is also considered, in which $\hat{\mu} = T^{-1}\sum_{t=1}^{T}\pi_t$.

By replacing $s_{1t}$ and $s_{2t}$ in $\pi_t$ the definition of the variance ratio $\hat{V}_{2k}$, equation (12), the following persistence measures occur, based in signals:

$$\hat{S}_{1k} = \left\{\frac{1}{(T-k+1)k}\sum_{t=k}^{T}(s_{1t}, \cdots, s_{1t-k+1})^2\right\}\Big/\left\{\frac{1}{T}\sum_{t=1}^{T}(s_{1t})^2\right\} \tag{17}$$

$$\hat{S}_{2k} = \left\{ \frac{1}{(T-k+1)k} \sum_{t=k}^{T} (s_{2t}, \cdots, s_{2t-k+1} - k\hat{\mu}_{s2})^2 \right\} \Big/ \left\{ \frac{1}{T} \sum_{t=1}^{T} (s_{2t} - \hat{\mu}_{s2})^2 \right\} \qquad (18)$$

where: $\hat{\mu}_{s2} = T^{-1} \sum_{t=1}^{T} s_{2t}$.

Here are presented some measures of the variance ratio presented in Cribari-Neto and Cassiano (2005).

## 4. ANALYSIS AND DISCUSSION

In Brazil, the inflation target was defined considering the variation of the Broad Consumer Price Index (IPCA) calculated by the Brazilian Institute of Geography and Statistics (IBGE). This index aims to measure price variations for households with income between 1 and 40 minimum wages. The choice of price indexes to consumer is common in most inflation targeting regimes, because it is considered an adequate measure to evaluate the evolution of the population's real income.

The adoption of the full IPCA in the country was due to two reasons. The first is that, although in the long term the core and the inflation tend to converge, in the short term, they may diverge significantly. The second, and maybe the most important, is a question of transparency and credibility. Therefore, since the variation of the IPCA is adopted as an official measurement of the Brazilian inflation, the present work aimed to analyze the projections concerning this variable, not only for horizons considered of short term (one step ahead) but also for long term (twelve steps ahead), of the several institutions that took part of the Focus-Market Report, from January 2002 to February 2007.

It is useful to point out that, although the research Focus/BACEN had, in February 2007, 74 participating institutions, according to the reference point adopted, there is an indicator of the presence of 151 institutions. But not all of them responded from the beginning (January/2002) and, besides, many of them do not keep a constant participation. With that, the institutions went in and out, besides those that did not respond regularly, leading to a consequent lack of information in some points of time.

From this point on, it was chosen to analyze and test the superior predictive ability only of the institutions which presented inflation projections, month by month (one step ahead) in the mentioned period, which implied in a sample with 20 institutions, namely: institutions 04, 10, 18, 22, 31, 37, 40, 41, 45, 57, 59, 68, 72, 78, 86, 110, 112, 113, 114, and 115, identified this way in the dataset sourced by the Gerin. Döpke, Dovern, Fritsche and Slacacek (2006) also used a sample of around 20-30 projections. The advantage of use Gerin survey data, as Roberts (1995), is that assumptions about the structure of the rest of economy are made unnecessary.

Similar analysis could not be done for twelve steps ahead because of the constant lack of information. The alternative adopted was to compare the projections of the following models: Focus/Bacen, Top5/Bacen, Bacen1 (central projection – reference scenery), Bacen2 (central projection – market scenery), AR(2), ARMA(3,1), *Random Walk*, Top5* and Focus20.

The 20 institutions mentioned above are called "Focus20", mentioned as part of a whole, named Focus/Bacen. It is important to point out that these are not the 20 institutions with the highest predictive power.

The Gerin recognizes the prediction excellence of the institutions which take part of the research on the market expectations by means of the *rankings Top* 5 of short, medium and long terms[10]. Here, however, the five best predictive institutions, according to selection criteria, are used, out of the 20 institutions previously selected. However, for comparison purposes, the reference

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[10] The Gerin, in the short term *ranking,* evaluates the precision of the estimative with one month of difference in relation to the indicator's edition, for the previous 6 months. The mid-term ranking considers the average precision of the projections of three consecutive periods of four months in comparison to the effective results of three months – the reference month and the two months that precede it. The long term *ranking* considers the precision of the projections informed in 12 months for the annual indicator published in the subsequent January.

dates follow the last working day, previously to the date when the IPCA-15[11] is released, as the Gerin does to identify the TOP 5 institutions.

Since the adoption of the regime inflation targeting, the effective inflation surpassed the tolerance interval in 2001, 2002 and 2003 (Table 1). In 2004 and 2005, the IPCA variation was above the center of the target, but inside the superior limit of the tolerance interval, while in 2006 and 2007, the inflation was below the center of the target, but inside the tolerance limit[12].

<div style="text-align:center">

| Table 1 about here |

</div>

For 2008, the median of the market expectations for the inflation, collected by the Gerin and published in Focus, Market Report of May 9th, 2008, was 4,96%, a value above the inflation target of 4,5%. Furthermore, it is possible to see a rising tendency. For 2009, it is expected that the inflation remains next to the target, which was determined in 4,5% (4, 47%).

Figure 1 presents the behavior of part of these inflation expectations, in the case of the 20 institutions selected (Focus20). It can be seen, in Figure 1, a high influence of the uncertainties of 2002 on the inflation expectations, led by the lack of confidence in the Brazilian economy, taking them to the highest levels of the period in case.

<div style="text-align:center">

| Figure 1 about here |

</div>

Next, it can be observed a certain convergence process of the inflation expectations for lower levels, as well as for the pre-defined inflation targets. Also, after the implicit economic recovery in these behaviors, it can be noticed the effect of the jump inflation on the expectations, in 2004.

Also, Figure 2 presents the path of the standard deviation and the variation coefficient[13] of the inflation expectations, either at complete (Focus) or partial (Focus20) sample levels, according to the behavior of the effective inflation, and the last one always presenting a scale in the first ordinate.

<div style="text-align:center">

| Figure 2 about here |

</div>

Figure 2 reveals that the divergence among expectations decreases as the regime inflation targeting goes by and this result is confirmed by the correlation coefficient between the standard deviation and the effective inflation[14], when it shows that the divergence increases (reduces) with the increase (fall) of the effective inflation. It can be seen in some points that the divergence tends

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[11] In May 2000, the IBGE also started to make the Broad Consumer Price Index-15 (IPCA-15) available on the internet, whose period of price collection is, approximately, from the 15th day of the previous month to the 15th day of the reference month.

[12] For further details about the causes of the failure in meeting the inflation targets, as well as the alternatives to fix the situation, consult the site of the Central Bank of Brazil in System of Targets for Inflation (http://www.bcb.gov.br/?SISMETAS) which, seeking more transparency, started to inform society about the conduction of the target regime through many means of communication.

[13] The effect of variation or dispersion in relation to the average can be measured by the relative dispersion, defined by: Relative Dispersion = Absolute Dispersion/Average. If the absolute dispersion is the standard deviation, the relative dispersion is called coefficient of variation (CV).

[14] **Correlation Coefficient**

|  | IPCA |
|---|---|
| Standard Deviation FOCUS20 | 0.66 ** |
|  | (6,805) |
| C.V. FOCUS20 | -0.26 * |
|  | (-2,086) |

**Source**: Data calculated by the authors; ( ) $t_o$ calculated statistics; *, ** It is rejected the null hypothesis of null correlation at 5% and 1% of significance, respectively.

to increase when the inflation changes abruptly for any direction: cases occurred in 11/2002 (IPCA increased in 1,71 p.p.) and 06/2003 (IPCA registered downfall of 0,76 p.p.).

As to the variation coefficient, which seeks to help in the inference on the variability of the inflation expectations, since the standard deviation is influenced by the data magnitude, it can be seen that it tends to increase mainly when deflation occurs: cases of 06/2003 (-0,15%), 06/2005 (-0,02%) and 06/2006 (-0,21%). This inverse relation between the variation coefficient and the effective inflation is observed in the correlation coefficient between them which indicates a negative linear association level.    The empirical results above corroborate those presented by Mankiw, Reis and Wolfers (2003), in other words, the divergence increases (reduces) with the increase (downfall) of the effective inflation and this, many times, tends to increase when inflation changes abruptly for any direction. It can still be noticed some difficulty for the Brazilian institutions in previewing deflations.

Before analyzing the problem of the superior predictive ability, Table 2 shows the institutions which present the highest predictive performance in an ascending order, according to the criteria of the Root Mean Squared Error (RMSE), Mean Squared Error (MSE) and Mean Absolute Error (MAE), this way allowing the proposal of *benchmark* "models", which is the case of the Top5*, formed by the median of the projections of the five best institutions.

Afterwards, the multiple comparison of projections is applied, according to Hansen (2005), in which the predictive power of the Top5* is evaluated, inclusively in relation to the Top5/BACEN, which may have suffered changes in its composition in the period studied[15].

Table 2 reveals the good performance of the institutions 10, 78, 37, 18 and 59, pointing out the institution 10. With that, there is only one Top 5* model formed by these institutions. For purposes of comparison with the other models, the prediction median of these institutions is used, just like the Gerin does in the release of the median of the Top 5 – short term.

| Table 2 about here |

The worst performance was presented by the institutions 45, 41 and 72. The *random walk* model, in which the best prediction for the inflation in the t+1 period would be the inflation in the t instant, presented lower results, compared to the others, according to the RMSE, MSE and MAE criteria. In this case, there are signs that the institutions are at least developing "models" with a predictive power superior to the very observation of the inflation that is one period out of step.

The Focus20 was also pointed out, demonstrating a better predictive performance than the Focus/Bacen itself[16], maybe revealing the fact that these institutions which presented their expectations in a constant way had a better monitoring of the prices and/or used more sophisticated means to forecast inflation. Or, it is simply due to a higher dispersion when the sample is widened. Besides, the Focus20 showed higher predictive ability in comparison to other institutions.

Table 3 shows long term predictive ability, still maintaining the same evaluation criteria. Previously, there could be an explanation on the models used in the comparison. As it can be seen, there was no comparison between the predictive capacity of the institutions individually, as it used to be, since the sample studied did not present enough information for such analysis. In this case, a choice was made for the median of the Top5*, the Focus/Bacen, the Top5/Bacen, the Focus20 and Bacen1, which indicates a central-scenario projection of reference, Bacen2, suggesting a market central scenario projection, *Random Walk*, AR(2) and ARMA(3, 1), maintaining the same reference point defined in the short term reference.

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[15] There is not a definite imposition that the TOP5* should be constant, an altered composition would also be possible along the period, since it is enough to make the introduction of new forecasts and then implement a new selection of the five best ones.

[16] Again, it is interesting to point out that the FOCUS20 refer to the median of the inflation expectations of the 20 institutions selected, while the FOCUS refers to the median of all the inflation expectations issued by the Gerin, evidently respecting the dates of reference for the purpose of comparison.

Although it is impossible to compare individually the predictive capacity of the institutions, this analysis is also relevant, since the information can be used as reference for other economic agents,  except the models proposed here (Top5* and Focus20). The AR(2) and ARMA (3,1) models, on the other hand, are considered due to their large use and they can easily become reference for some economic agents.

The empirical results now show that the largest predictive capacities are in the AR(2) and ARMA (3,1) models selected. The highest predictive power of the *random walk* is remarkable, in comparison to the projections of the market and Bacen here considered.

So, when following the criteria of RMSE, MSE and MAE, which are very much used, the empiric results show the existence of heterogeneity in the predictive capacity of the Brazilian inflation, diverging from the results of the simple equivalence tests presented in Table 3. Based on this information, it is possible to imagine that the agents would form new prices from projections different from the inflation.  If these projections are significant for the determination of prices, then such divergence may produce changes in the intergroup and intergroup relative prices.

In the next subsection, a more powerful statistical test is applied, the SPA test of Hansen (2005), to verify the superior predictive ability, allowing a stronger corroboration of the problem of disagreement about inflation expectations, because, if  institutions with superior predictive power exist, then the divergence is statistically reaffirmed in the presence of co-movement among the projections.

For such, it was used the MULCOM (*Multiple Comparison*) package version 1.00, which can be used interactively together with the OxPack for OxMetrics[17].

## 4.1. Superior Predictive Ability test (SPA) and disagreement

The analysis of the results of the SPA test can be divided into three parts:

 The first one contains the descriptive statistics, such as the number of competing models, the size of the sample, bootstrap parameters, among others. The second one informs about model performance, and six pair-wise comparisons. The six models compared to the *benchmark* model are: (1) The "Most Significant" model, whose performance is the most "significant" relative to the *benchmark* model in the sample analyzed; and (2)-(6) are those models with a performance that corresponded to the 75%, 50% (median), 25% and 0%. The performance is measured in terms of specified loss function. In the present case, the MSE is the measure considered. The *t-stat* indicates the significance of the model in comparison to its performances compared with the *benchmark* model. But it is important to notice that this statistic is not t-distributed, and the "p-value" reported next to it cannot be interpreted as a p-value, rather it is a number that is calculated like a p-value. The third part offers information to verify if the hypothesis that the *benchmark* model is the best forecasting model, in a certain level of significance.

For the short term analysis, the investigation starts with the use of *benchmark* models considered weak for inflation prediction. It would be the case of the *random walk* and the institutions that had the worst predictive performance, according to Table 2, in other words, the institutions 45, 41 and 72. With these models, it is sought to corroborate their predictive inferiority, which would already confirm the existence of disagreement among the inflation expectations.

Other four models served as *benchmark*: Top 5/Bacen, Top5*, Focus/Bacen and Focus20. In these cases, it is investigated if it is reasonable to consider that the Focus/Top5 surveys, for example, carried out by the Bacen and that may be adopted in the market, are good predictors of the inflation rate in a short term prediction horizon. Another objective was to verify if these inflation projections have a greater predictive ability than some unvaried linear models AR(1), SA (1), ARMA(2,1) and SARMA(5,3). For the long term analysis, the rationale is the same, but, due to the

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[17] Hansen and Lunde (2007) and Doornik and Hendry (2006).

reasons already commented, only four models are considered as *benchmark*: *random walk*, AR(2), Top5/Bacen and Focus/Bacen.

The empiric results for the short term cases are presented in Tables 4 and 5, divided in panels, while the long term cases may be seen in Table 6.

The panels A-D, Table 4, show *SPA-Consistent p-values*, suggesting that the b*enchmark* models, in this case, the *random walk* and the institutions 41, 45 and 72, respectively, are surpassed at least for one of the models of the other institutions, thus confirming the existence of a disagreement in the Brazilian inflation expectations.

| Table 4 about here |

On the other hand, the results presented in the panels E-H showed, Table 5: one *SPA-Consistent p-value* of 0.8619, demonstrating that the Top5/Bacen is not inferior to the other models, except for the Top5*, which is not included in this test; one *SPA-Consistent p-value* of 0.9996, revealing that the Top5* would not also be inferior to the others, including the Top5/BACEN itself; and, finally, the inferiority, not only of the Focus/Bacen but also of the Focus20, since in a reasonable level of significance, they would be surpassed by at least one of the alternatives.

| Table 5 about here |

The panels I-L, Table 6, showed a *SPA-Consistent p-value* of 0.02470, suggesting that the *random walk* model, in the long term analysis, is surpassed by at least one of the alternative models, but that the AR(2) would not be surpassed by the others (*SPA-Consistent p-value* de 0.70). It is still possible to observe that the Top5/Bacen and Focus/Bacen would be inferior to the other models in the prediction of the twelve steps ahead.

| Table 6 about here |

In short, both short and long term analyses offer empiric results favorable to the existence of disagreement about inflation expectations, in which some models/institutions exert superior predictive ability, corroborating the results presented in Mankiw, Reis and Wolfers (2003). They are also in accordance with the results of Lima and Cespedes (2003), when these authors say that simple autoregressive models are able to generate more exact long term predictions, but do not corroborate their results that the market expectations are predictions for inflation statistically similar to those of the Bacen.

It can also be observed that, if the institutions present not only divergences in the projections of the Brazilian inflation, but also some degree of inertia in the dynamics of their expectations. For such, the variance ratio test is applied, a measure systematically used for the measurement of the persistence of long term shock.

The presence of certain degree of inertia in the expectations would indicate that the adjustment of these expectations would gradually occur, according to the shocks in the results become stronger with the estimates, but they are not clear as to the institutions 41 and determinants, such as Selic interest rate, effective inflation, among others.


## 4.2. Persistence and disagreement

Table 7 shows the estimate of the persistence measures $\hat{V}_{1k}$, $\hat{V}_{2k}$, $\hat{S}_{1k}$, and $\hat{S}_{2k}$ for the inflation expectations, one step ahead, of the institutions that presented the best and the worst

predictive performances, according to Table 2. In all the case is observed that $\hat{V}_{1k}$ and $\hat{V}_{2k}$

decreases as the value of $k$ increases and presents similar values. For $k = 2$ and $k = 3$, these measures suggest sharp shock effects, but the estimates decrease, so that $k = 24$, namely, 24 months or 2 years after the initial shock $\hat{V}_{1k}$ and $\hat{V}_{2k}$ remain next to 0.20 and 0.10, with some cases a little above or a little below these values.

$$\boxed{\text{Table 7 about here}}$$

Supported by these results, it can be said that the inertia in the dynamics of the expectations of these institutions are of second order. In this sense, shock effects must be quickly dissipated.

Such 72, because they present contradictory estimates $\hat{S}_{1k}$ and $\hat{S}_{2k}$. The last institutions mentioned presented weak predictive performances.

In general, it is possible to say that the measures of estimated persistence suggest sharp shock effects, but that such effects decrease as time passes, for most of the cases investigated.

## 5. CONCLUSION

Inflation expectations are considered a key-variable in the specification of macroeconomic models, in which such relevance comes from Friedman (1968) and Phelps (1969). Recently, the expectations, especially those of inflation, have received great interest in the discussion about the regime inflation targeting and the informational rigidity that would lead to price rigidity. Therefore, the present work aimed to offer literature the results of the Brazilian dataset, exploring the problem of disagreement about inflation expectations. To achieve this, it evaluates the predictive ability of the institutions that took part in the research "Focus-Market Report" of the Gerin/Bacen in the period from January 2002 to February 2007, starting from a multiple comparison of predictions of a certain *benchmark* model via test for superior predictive ability proposed by Hansen (2005) – test SPA.

The empiric results reinforce the statement that the divergence in the expectations increases (reduces) with the increase (fall) of the effective inflation and that it sometimes tends to increase when inflation changes abruptly for any direction, according to the result presented in Mankiw, Reis and Wolfers (2003). But the existence of disagreement about expectations becomes stronger with its corroboration via RMSE, MSE e MAE criteria and, mainly, with the application of the SPA statistical test in short and long term horizons.

Some of the results achieved take the same direction of Lima and Cespedes (2003) when they show that simple autoregressive models are able to generate more exact long term predictions, but other authors do not corroborate their result that market expectations are predictions for the inflation statistically similar to those of the Bacen. It can also be said, based on the estimated persistence measures, that there is a certain degree of inertia in the expectation dynamics, in which the sharp shock effects occur, but that such effects decrease as time goes by, for most part of the cases analyzed.

So, there would be, besides disagreement in the expectations, a co-movement among the series and a certain degree of inertia, indicating that their adjustment would gradually occur, due to shocks in their determinants, such as the Selic interest rate, effective inflation, among others, depending on the speed of the shock effect fall, disorganized and in the same direction. Consequently, the presence of rigidity in the costs would be inevitable, because they do not adjust coordinately, and they may not automatically adjust when shocks occur, due to the informational rigidity problem, for example.

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**Attachment – Figures and Tables**
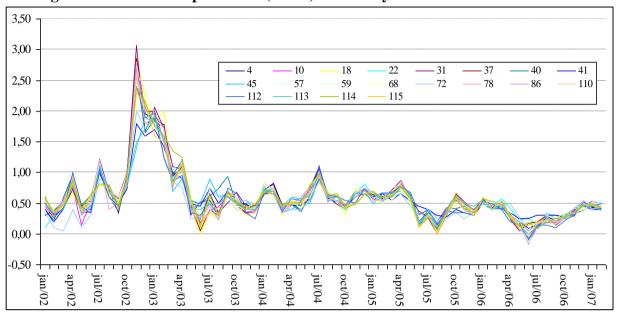**Figure 1 – Inflation Expectations (IPCA) – Monthly from 01/2002 to 02/2007**



**Figure 2 – Standard Deviation. Variation Coefficient (FOCUS/FOCUS20) and Observed Inflation (IPCA)**
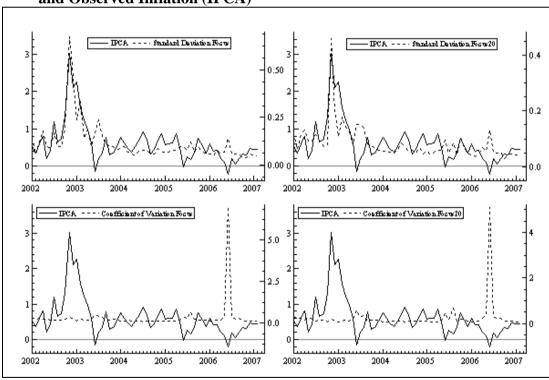


**Table 1 – Report of the Targets for Inflation in Brazil**

| Year | Norm | Date | Target (%) | Band (p.p) | Inferior and Superior Limits (%) | Effective inflation (IPCA % a.a.) |
|------|------|------|-----------|-----------|----------------------------------|-----------------------------------|
| 1999 | Resolução 2.615 | 30/6/1999 | 8 | 2 | 6-10 | 8.94 |
| 2000 | | | 6 | 2 | 4-8 | 5.97 |
| 2001 | | | 4 | 2 | 2-6 | 7.67 |
| 2002 | Resolução 2.744 | 28/6/2000 | 3.5 | 2 | 1.5-5.5 | 12.53 |

| 2003* | Resolução 2.842 | 28/6/2001 | 3.25 | 2 | 1.25-5.25 | 9.30 |
| | Resolução 2.972 | 27/6/2002 | 4 | 2.5 | 1.5-6.5 | |
| 2004* | Resolução 2.972 | 27/6/2002 | 3.75 | 2.5 | 1.25-6.25 | 7.60 |
| | Resolução 3.108 | 25/6/2003 | 5.5 | 2.5 | 3-8 | |
| 2005 | Resolução 3.108 | 25/6/2003 | 4.5 | 2.5 | 2-7 | 5.69 |
| 2006 | Resolução 3.210 | 30/6/2004 | 4.5 | 2 | 2.5-6.5 | 3.14 |
| 2007 | Resolução 3.291 | 23/6/2005 | 4.5 | 2 | 2.5-6.5 | 4.46 |
| 2008 | Resolução 3.378 | 29/6/2006 | 4.5 | 2 | 2.5-6.5 | |
| 2009 | Resolução 3.463 | 26/6/2007 | 4.5 | 2 | 2.5-6.5 | |

Source: BCB. * The Open Letter, of 10/21/2003, established adjusted targets of 8,5% for 2003 and 5,5% for 2004.

**Table 2 – MAE, MSE, AND RMSE (in ascending order - *Ranking*)**

| Model | MAE | Model | MSE | Model | RMSE |
|---|---|---|---|---|---|
| Top5* | 0.091 | Top5* | 0.015 | Top5* | 0.123 |
| 10 | 0.096 | 10 | 0.017 | 10 | 0.132 |
| 78 | 0.100 | 78 | 0.019 | 78 | 0.137 |
| 37 | 0.101 | Top5/BACEN | 0.019 | Top5/BACEN | 0.137 |
| Top5/BACEN | 0.104 | 37 | 0.019 | 37 | 0.139 |
| Focus20 | 0.109 | 18 | 0.022 | 18 | 0.148 |
| 18 | 0.113 | 59 | 0.023 | 59 | 0.152 |
| 59 | 0.114 | Focus20 | 0.024 | Focus20 | 0.155 |
| 110 | 0.115 | 110 | 0.025 | 110 | 0.159 |
| 68 | 0.116 | 4 | 0.026 | 4 | 0.162 |
| 57 | 0.116 | 68 | 0.026 | 68 | 0.163 |
| 31 | 0.120 | 86 | 0.029 | 86 | 0.170 |
| 4 | 0.121 | 31 | 0.030 | 31 | 0.174 |
| 22 | 0.128 | 57 | 0.033 | 57 | 0.181 |
| 114 | 0.129 | 114 | 0.035 | 114 | 0.187 |
| Focus/BACEN | 0.129 | 22 | 0.036 | 22 | 0.189 |
| 86 | 0.129 | Focus/BACEN | 0.039 | Focus/BACEN | 0.197 |
| 115 | 0.132 | 112 | 0.039 | 112 | 0.198 |
| 40 | 0.144 | 40 | 0.040 | 40 | 0.200 |
| 112 | 0.147 | 115 | 0.040 | 115 | 0.200 |
| 113 | 0.148 | 113 | 0.067 | 113 | 0.258 |
| 45 | 0.162 | 72 | 0.069 | 72 | 0.263 |
| 41 | 0.177 | 41 | 0.072 | 41 | 0.267 |
| 72 | 0.194 | 45 | 0.083 | 45 | 0.288 |
| SAR(1) | 0.239 | AR(1) | 0.132 | AR(1) | 0.363 |
| AR(1) | 0.241 | SAR(1) | 0.137 | SAR(1) | 0.370 |
| RW | 0.282 | RW | 0.152 | RW | 0.390 |
| SARMA(5.3) | 0.342 | ARMA(2.1) | 0.215 | ARMA(2.1) | 0.464 |
| ARMA(2.1) | 0.348 | SARMA(5.3) | 0.218 | SARMA(5.3) | 0.467 |

Source: BACEN; IBGE. Values calculated by the authors. The ARMA(p.q) models were estimated using EViews 6. and chosen through the Schwarz selection criterion.

**Table 3 – MAE. MSE. AND RMSE (in ascending order - *Ranking*)**

| Model | MAE | Model | MSE | Model | RMSE |
|---|---|---|---|---|---|
| AR(2) | 0.381 | AR(2) | 0.320 | AR(2) | 0.566 |
| ARMA(3.1) | 0.409 | ARMA(3.1) | 0.350 | ARMA(3.1) | 0.592 |
| RW | 0.556 | RW | 0.679 | RW | 0.824 |
| TopFive/BACEN | 2.224 | TopFive/BACEN | 10.219 | TopFive/BACEN | 3.197 |
| Focus20 | 2.419 | Focus20 | 11.982 | Focus20 | 3.462 |
| Focus/BACEN | 2.463 | Focus/BACEN | 12.673 | Focus/BACEN | 3.560 |
| Focus5* | 2.589 | Focus5 | 14.999 | Focus5 | 3.873 |
| BACEN 2 | 3.367 | BACEN 2 | 22.784 | BACEN 2 | 4.773 |
| BACEN 1 | 3.880 | BACEN 1 | 24.881 | BACEN 1 | 4.988 |

Source: BACEN; IBGE. Values calculated by the authors. The AR(p) and ARMA(p.q) models were estimated using Eviews 6. and chosen through the Schwarz selection criterion. P.S..: BACEN1 implies in the central scenario projection of reference and BACEN2 in the central scenario projection of market. both presented in the "Report of Inflation". of the BACEN.

**Table 4 – Tests for Superior Predictive Ability (Short Term)**

| Panel A: Results for the Benchmark *Random Walk* | "Model" | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|
| Evaluated by MSE | Benchmark: RW | 0.15175 | -- | -- |
| m = 20 (number of models) | Most Significant | 113 | 0.06650 | 3.79073 | 0.00060 |
| n = 62 (sample size) | Best | 10 | 0.01734 | 2.48782 | 0.01580 |
| B = 10. 000 (resamples) | Model_25% | 110 | 0.02516 | 2.48531 | 0.01570 |
| q = 0.5 (dependence) | Median | 57 | 0.03277 | 2.74673 | 0.00940 |
| | Model_75% | 40 | 0.03987 | 2.20649 | 0.02660 |
| | Worst | 45 | 0.08306 | 3.21059 | 0.00210 |
| | SPA p-values: | | $SPA_{Lower}$ | $SPA_{Consistent}$ | $SPA_{Upper}$ |
| | | | 0.00200 | 0.00200 | 0.00200 |

| Panel B: Results for the Benchmark 41 | "Model" | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|
| Evaluated by MSE | Benchmark: 41 | 0.07153 | -- | -- |
| m = 19 (number of models) | Most Significant | 57 | 0.03277 | 2.45947 | 0.01420 |
| n = 62 (sample size) | Best | 10 | 0.01734 | 2.07410 | 0.03460 |
| B = 10. 000 (resamples) | Model_25% | 110 | 0.02516 | 1.99207 | 0.03810 |
| q = 0.5 (dependence) | Median | 31 | 0.03034 | 1.54431 | 0.07510 |
| | Model_75% | 40 | 0.03987 | 1.38385 | 0.08900 |
| | Worst | 45 | 0.08306 | -0.64552 | 0.74910 |
| | SPA p-values: | | $SPA_{Lower}$ | $SPA_{Consistent}$ | $SPA_{Upper}$ |
| | | | 0.03240 | 0.03250 | 0.03250 |

| Panel C: Results for the Benchmark 45 | "Model" | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|
| Evaluated by MSE | Benchmark: 45 | 0.08306 | -- | -- |
| m = 19 (number of models) | Most Significant | 113 | 0.06650 | 1.79606 | 0.05110 |
| n = 62 (sample size) | Best | 10 | 0.01734 | 1.56794 | 0.07690 |
| B = 10. 000 (resamples) | Model_25% | 110 | 0.02516 | 1.47945 | 0.08450 |
| q = 0.5 (dependence) | Median | 31 | 0.03034 | 1.25139 | 0.09340 |
| | Model_75% | 40 | 0.03987 | 1.12294 | 0.12020 |
| | Worst | 41 | 0.07153 | 0.64552 | 0.25090 |
| | SPA p-values: | | $SPA_{Lower}$ | $SPA_{Consistent}$ | $SPA_{Upper}$ |
| | | | 0.10830 | 0.10830 | 0.10830 |

| Panel D: Results for the Benchmark 72 | "Model" | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|
| Evaluated by MSE | Benchmark: 72 | 0.06917 | -- | -- |
| m = 19 (number of models) | Most Significant | 57 | 0.03277 | 3.98665 | 0.00010 |
| n = 62 (sample size) | Best | 10 | 0.01734 | 2.93896 | 0.00570 |
| B = 10. 000 (resamples) | Model_25% | 110 | 0.02516 | 2.91920 | 0.00580 |
| q = 0.5 (dependence) | Median | 31 | 0.03034 | 2.09497 | 0.03020 |
| | Model_75% | 40 | 0.03987 | 1.95368 | 0.03860 |
| | Worst | 45 | 0.08306 | -0.51415 | 0.71910 |
| | SPA p-values: | | $SPA_{Lower}$ | $SPA_{Consistent}$ | $SPA_{Upper}$ |
| | | | 0.00140 | 0.00140 | 0.00140 |

Obs.: Values calculated by the authors using the OxEdit.

**Table 5 – Tests for Superior Predictive Ability (Short Term)**

| Panel E: Results for the Benchmark Top 5 | "Model" | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|
| Evaluated by MSE | Benchmark: Top 5 | 0.01881 | -- | -- |
| m = 27 (number of models) | Most Significant | 10 | 0.01734 | 0.86760 | 0.18500 |
| n = 62 (sample size) | Best | 10 | 0.01734 | 0.86760 | 0.18500 |
| B = 10. 000 (resamples) | Model_25% | 04 | 0.02619 | -1.70405 | 0.95080 |
| q = 0.5 (dependence) | Median | 22 | 0.03555 | -2.50679 | 0.99010 |
| | Model_75% | 41 | 0.07153 | -2.03848 | 0.96250 |
| | Worst | SARMA | 0.21820 | -3.00996 | 0.99440 |
| | SPA p-values: | | $SPA_{Lower}$ | $SPA_{Consistent}$ | $SPA_{Upper}$ |
| | | | 0.44500 | 0.86190 | 0.91010 |

| Panel F: Results for the Benchmark Top5* | "Model" | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|
| | | "Model" | Sample Loss | t-statistic | "p-value" |
| m = 28 (number of models) | Most Significant | 10 | 0.01734 | -1.01752 | 0.84450 |
| n = 62 (sample size) | Best | 10 | 0.01734 | -1.01752 | 0.84450 |
| B = 10. 000 (resamples) | Model_25% | 110 | 0.02516 | -1.92955 | 0.95730 |
| q = 0.5 (dependence) | Median | 22 | 0.03555 | -2.76910 | 0.99370 |

| | | | | | |
|---|---|---|---|---|---|
| | Model_75% | 72 | 0.06917 | -2.92735 | 0.99360 |
| | Worst | SARMA | 0.21820 | -3.00281 | 0.99440 |

| SPA p-values: | | | SPA$_{Lower}$ | SPA$_{Consistent}$ | SPA$_{Upper}$ |
|---|---|---|---|---|---|
| | | | 0.92870 | 0.99960 | 1.00000 |

**Panel G: Results for the Benchmark Focus** "Model" Sample Loss t-statistic "p-value"

| | "Model" | | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|---|
| Evaluated by MSE | Benchmark: | Focus | 0.03862 | -- | -- |
| m = 28 (number of models) | Most Significant | Top5* | 0.01521 | 1.98268 | 0.04030 |
| n = 62 (sample size) | Best | Top5* | 0.01521 | 1.98268 | 0.04030 |
| B = 10. 000 (resamples) | Model_25% | Focus20 | 0.02397 | 1.75596 | 0.06130 |
| q = 0.5 (dependence) | Median | 114 | 0.03487 | 0.53356 | 0.28250 |
| | Model_75% | 72 | 0.06917 | -3.47075 | 0.99870 |
| | Worst | SARMA | 0.21820 | -3.14314 | 0.99620 |

| SPA p-values: | | | SPA$_{Lower}$ | SPA$_{Consistent}$ | SPA$_{Upper}$ |
|---|---|---|---|---|---|
| | | | 0.12120 | 0.12620 | 0.14230 |

**Panel H: Results for the Benchmark Focus20** "Model" Sample Loss t-statistic "p-value"

| | "Model" | | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|---|
| Evaluated by MSE | Benchmark: | Focus20 | 0.02397 | -- | -- |
| m = 28 (number of models) | Most Significant | Top5* | 0.01521 | 2.43416 | 0.01570 |
| n = 62 (sample size) | Best | Top5* | 0.01521 | 2.43416 | 0.01570 |
| B = 10. 000 (resamples) | Model_25% | 110 | 0.02516 | -0.36423 | 0.65930 |
| q = 0.5 (dependence) | Median | 22 | 0.03555 | -2.20945 | 0.98020 |
| | Model_75% | 72 | 0.06917 | -2.95053 | 0.99420 |
| | Worst | SARMA | 0.21820 | -3.01386 | 0.99430 |

| SPA p-values: | | | SPA$_{Lower}$ | SPA$_{Consistent}$ | SPA$_{Upper}$ |
|---|---|---|---|---|---|
| | | | 0.04580 | 0.06730 | 0.07610 |

Obs.: Values calculated by the authors using the OxEdit.

## Table 6 – Tests for Superior Predictive Ability (Long Term)

**Panel I: Results for the Benchmark *Random Walk*** "Model" Sample Loss t-statistic "p-value"

| | "Model" | | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|---|
| Evaluated by MSE | Benchmark: | RW | 0.67871 | -- | -- |
| m = 8 (number of models) | Most Significant | AR | 0.31984 | 2.37776 | 0.01860 |
| n = 60 (sample size) | Best | AR | 0.31984 | 2.37776 | 0.01860 |
| B = 10. 000 (resamples) | Model_25% | TopFive | 10.21928 | -2.38843 | 0.97910 |
| q = 0.5 (dependence) | Median | Focus | 12.67272 | -2.47603 | 0.98310 |
| | Model_75% | Top5* | 14.99866 | -2.41508 | 0.98130 |
| | Worst | BACEN1 | 24.88102 | -3.06999 | 0.99500 |

| SPA p-values: | | | SPA$_{Lower}$ | SPA$_{Consistent}$ | SPA$_{Upper}$ |
|---|---|---|---|---|---|
| | | | 0.02470 | 0.02470 | 0.02470 |

**Panel J: Results for the Benchmark AR(2)** "Model" Sample Loss t-statistic "p-value"

| | "Model" | | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|---|
| Evaluated by MSE | Benchmark: | AR | 0.31984 | -- | -- |
| m = 8 (number of models) | Most Significant | ARMA | 0.35043 | -0.50590 | 0.69590 |
| n = 60 (sample size) | Best | ARMA | 0.35043 | -0.50590 | 0.69590 |
| B = 10. 000 (resamples) | Model_25% | TopFive | 10.21928 | -2.46717 | 0.98190 |
| q = 0.5 (dependence) | Median | Focus | 12.67272 | -2.53833 | 0.98470 |
| | Model_75% | Top5* | 14.99866 | -2.46490 | 0.98280 |
| | Worst | BACEN1 | 24.88102 | -3.09904 | 0.99520 |

| SPA p-values: | | | SPA$_{Lower}$ | SPA$_{Consistent}$ | SPA$_{Upper}$ |
|---|---|---|---|---|---|
| | | | 0.50340 | 0.70000 | 0.96740 |

**Panel K: Results for the Benchmark TopFive** "Model" Sample Loss t-statistic "p-value"

| | "Model" | | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|---|
| Evaluated by MSE | Benchmark: | TopFive | 10.21928 | -- | -- |
| m = 8 (number of models) | Most Significant | ARMA | 0.35043 | 2.47804 | 0.01790 |
| n = 60 (sample size) | Best | AR | 0.31984 | 2.46717 | 0.01810 |
| B = 10. 000 (resamples) | Model_25% | RW | 0.67871 | 2.38843 | 0.02090 |
| q = 0.5 (dependence) | Median | Focus | 12.67272 | -2.26885 | 0.97540 |
| | Model_75% | Top5* | 14.99866 | -2.03633 | 0.96280 |
| | Worst | BACEN1 | 24.88102 | -3.36574 | 0.99780 |

| SPA p-values: | | | SPA$_{Lower}$ | SPA$_{Consistent}$ | SPA$_{Upper}$ |
|---|---|---|---|---|---|
| | | | 0.01860 | 0.01860 | 0.01890 |

**Panel L: Results for the Benchmark Focus** "Model" Sample Loss t-statistic "p-value"

| | "Model" | | Sample Loss | t-statistic | "p-value" |
|---|---|---|---|---|---|
| Evaluated by MSE | Benchmark: | Focus | 12.67272 | -- | -- |
| m = 8 (number of models) | Most Significant | ARMA | 0.35043 | 2.54877 | 0.01510 |
| n = 60 (sample size) | Best | AR | 0.31984 | 2.53833 | 0.01530 |

| | | | | | |
|---|---|---|---|---|---|
| B = 10. 000 (resamples) | Model_25% | RW | 0.67871 | 2.47603 | 0.01690 |
| q = 0.5 (dependence) | Median | Focus20 | 11.98228 | 1.80776 | 0.05240 |
| | Model_75% | Top5* | 14.99866 | -1.76487 | 0.94420 |
| | Worst | BACEN1 | 24.88102 | -3.59028 | 0.99880 |
| | SPA p-values: | | $SPA_{Lower}$ | $SPA_{Consistent}$ | $SPA_{Upper}$ |
| | | | 0.03050 | 0.03050 | 0.03070 |

Obs.: Values calculated by the authors using the OxEdit.

**Table 7 – Persistence Measures**

| | K | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Vr | 0.91 | 0.74 | 0.64 | 0.52 | 0.44 | 0.41 | 0.36 | 0.35 | 0.30 | 0.28 | 0.28 | 0.28 | 0.26 |
| | vr2 | 0.91 | 0.72 | 0.63 | 0.50 | 0.41 | 0.27 | 0.19 | 0.18 | 0.14 | 0.13 | 0.14 | 0.14 | 0.12 |
| | S1 | 0.88 | 0.59 | 0.59 | 0.59 | 0.52 | 0.58 | 0.60 | 0.65 | 0.69 | 0.71 | 0.78 | 0.79 | 0.77 |
| Focus | S2 | 0.89 | 0.58 | 0.54 | 0.50 | 0.39 | 0.41 | 0.40 | 0.42 | 0.44 | 0.42 | 0.46 | 0.46 | 0.42 |
| | Vr | 0.89 | 0.73 | 0.51 | 0.44 | 0.39 | 0.34 | 0.32 | 0.31 | 0.26 | 0.25 | 0.24 | 0.24 | 0.22 |
| | vr2 | 0.89 | 0.72 | 0.50 | 0.42 | 0.36 | 0.25 | 0.18 | 0.17 | 0.14 | 0.12 | 0.12 | 0.12 | 0.10 |
| | S1 | 0.74 | 0.45 | 0.34 | 0.28 | 0.23 | 0.22 | 0.19 | 0.18 | 0.17 | 0.16 | 0.19 | 0.19 | 0.18 |
| 45 | S2 | 0.78 | 0.58 | 0.51 | 0.48 | 0.43 | 0.45 | 0.45 | 0.42 | 0.36 | 0.30 | 0.27 | 0.24 | 0.21 |
| | Vr | 0.92 | 0.71 | 0.69 | 0.56 | 0.44 | 0.41 | 0.35 | 0.32 | 0.28 | 0.26 | 0.26 | 0.26 | 0.24 |
| | vr2 | 0.92 | 0.70 | 0.68 | 0.54 | 0.41 | 0.28 | 0.19 | 0.17 | 0.14 | 0.12 | 0.13 | 0.13 | 0.11 |
| | S1 | 1.01 | 0.66 | 0.66 | 0.62 | 0.47 | 0.50 | 0.41 | 0.36 | 0.32 | 0.25 | 0.27 | 0.27 | 0.24 |
| 10 | S2 | 1.08 | 0.82 | 0.84 | 0.77 | 0.58 | 0.60 | 0.51 | 0.44 | 0.36 | 0.27 | 0.27 | 0.25 | 0.21 |
| | Vr | 0.84 | 0.70 | 0.63 | 0.51 | 0.38 | 0.34 | 0.28 | 0.27 | 0.24 | 0.22 | 0.22 | 0.21 | 0.19 |
| | vr2 | 0.84 | 0.70 | 0.62 | 0.50 | 0.37 | 0.24 | 0.15 | 0.15 | 0.13 | 0.11 | 0.11 | 0.11 | 0.08 |
| | S1 | 0.81 | 0.49 | 0.47 | 0.44 | 0.33 | 0.35 | 0.33 | 0.35 | 0.39 | 0.42 | 0.48 | 0.51 | 0.50 |
| 18 | S2 | 0.82 | 0.49 | 0.45 | 0.41 | 0.28 | 0.28 | 0.25 | 0.25 | 0.28 | 0.29 | 0.34 | 0.36 | 0.34 |
| | vr | 0.90 | 0.66 | 0.60 | 0.48 | 0.38 | 0.35 | 0.29 | 0.27 | 0.24 | 0.21 | 0.22 | 0.21 | 0.19 |
| | vr2 | 0.89 | 0.66 | 0.60 | 0.47 | 0.36 | 0.24 | 0.16 | 0.15 | 0.12 | 0.10 | 0.12 | 0.11 | 0.09 |
| | S1 | 0.94 | 0.56 | 0.53 | 0.47 | 0.38 | 0.40 | 0.32 | 0.31 | 0.26 | 0.20 | 0.21 | 0.17 | 0.13 |
| 37 | S2 | 0.95 | 0.69 | 0.61 | 0.54 | 0.43 | 0.47 | 0.39 | 0.34 | 0.27 | 0.20 | 0.19 | 0.16 | 0.11 |
| | vr | 0.89 | 0.70 | 0.61 | 0.52 | 0.47 | 0.43 | 0.39 | 0.37 | 0.33 | 0.31 | 0.31 | 0.30 | 0.28 |
| | vr2 | 0.88 | 0.69 | 0.60 | 0.49 | 0.45 | 0.30 | 0.21 | 0.19 | 0.15 | 0.14 | 0.15 | 0.14 | 0.12 |
| | S1 | 0.88 | 0.80 | 0.88 | 0.97 | 1.12 | 1.32 | 1.47 | 1.62 | 1.72 | 1.84 | 1.93 | 1.93 | 1.93 |
| 41 | S2 | 0.81 | 0.55 | 0.44 | 0.36 | 0.33 | 0.36 | 0.30 | 0.26 | 0.22 | 0.20 | 0.17 | 0.13 | 0.10 |
| | vr | 0.98 | 0.75 | 0.70 | 0.54 | 0.41 | 0.37 | 0.32 | 0.30 | 0.25 | 0.24 | 0.24 | 0.24 | 0.21 |
| | vr2 | 0.97 | 0.74 | 0.69 | 0.53 | 0.39 | 0.26 | 0.18 | 0.17 | 0.13 | 0.12 | 0.13 | 0.12 | 0.10 |
| | S1 | 0.88 | 0.50 | 0.46 | 0.45 | 0.37 | 0.37 | 0.33 | 0.31 | 0.26 | 0.22 | 0.22 | 0.21 | 0.18 |
| 59 | S2 | 0.89 | 0.51 | 0.45 | 0.43 | 0.34 | 0.33 | 0.29 | 0.26 | 0.21 | 0.17 | 0.17 | 0.16 | 0.13 |
| | vr | 0.95 | 0.82 | 0.72 | 0.61 | 0.53 | 0.41 | 0.39 | 0.38 | 0.33 | 0.31 | 0.32 | 0.31 | 0.29 |
| | vr2 | 0.94 | 0.81 | 0.70 | 0.58 | 0.49 | 0.28 | 0.20 | 0.20 | 0.17 | 0.15 | 0.16 | 0.16 | 0.14 |
| | S1 | 0.81 | 0.76 | 0.73 | 0.75 | 0.81 | 0.86 | 0.92 | 0.95 | 0.99 | 1.05 | 1.11 | 1.13 | 1.16 |
| 72 | S2 | 0.80 | 0.71 | 0.60 | 0.54 | 0.53 | 0.50 | 0.46 | 0.39 | 0.34 | 0.32 | 0.29 | 0.29 | 0.28 |
| | vr | 0.85 | 0.67 | 0.59 | 0.46 | 0.36 | 0.35 | 0.30 | 0.29 | 0.26 | 0.23 | 0.24 | 0.23 | 0.21 |
| | vr2 | 0.85 | 0.66 | 0.58 | 0.44 | 0.34 | 0.23 | 0.16 | 0.15 | 0.13 | 0.11 | 0.12 | 0.11 | 0.10 |
| | S1 | 0.88 | 0.66 | 0.63 | 0.63 | 0.51 | 0.52 | 0.53 | 0.55 | 0.58 | 0.57 | 0.62 | 0.64 | 0.62 |
| 78 | S2 | 0.89 | 0.66 | 0.62 | 0.61 | 0.46 | 0.44 | 0.43 | 0.44 | 0.46 | 0.43 | 0.47 | 0.47 | 0.45 |