

Eletromagnetismo- CEL065

«Formulário de Cálculo Vetorial & Eletromagnetismo»

Vetores unitários

Coordenadas retangulares – $\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$

Coordenadas cilíndricas – $\mathbf{a}_\rho, \mathbf{a}_\phi, \mathbf{a}_z$

Coordenadas esféricas – $\mathbf{a}_r, \mathbf{a}_\theta, \mathbf{a}_\phi$

Transformação de coordenadas

$$x = \rho \cos \phi = r \sin \theta \cos \phi$$

$$y = \rho \sin \phi = r \sin \theta \sin \phi$$

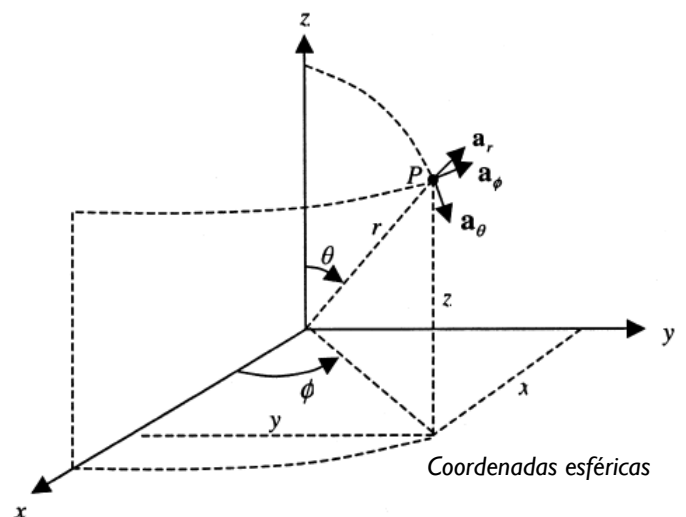
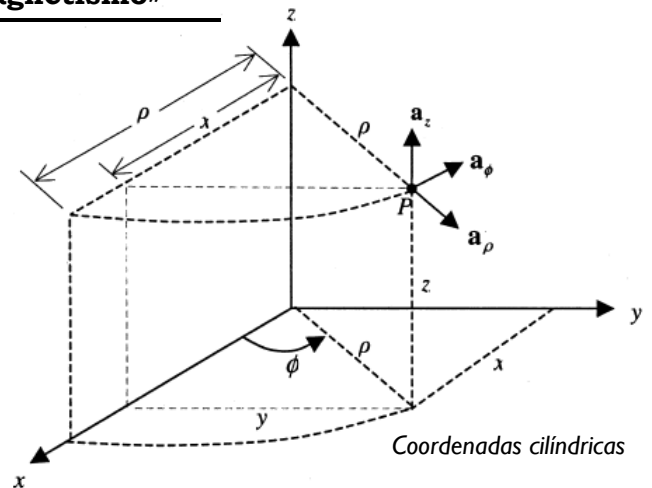
$$z = r \cos \theta$$

$$\rho = \sqrt{x^2 + y^2} = r \sin \theta$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{\rho^2 + z^2}$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right) = \tan^{-1} \left(\frac{\rho}{z} \right)$$



Transformação de coordenadas de componentes vetoriais

$$A_x = A_\rho \cos \phi - A_\phi \sin \phi = A_r \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$$

$$A_y = A_\rho \sin \phi + A_\phi \cos \phi = A_r \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$$

$$A_z = A_r \cos \theta - A_\theta \sin \theta$$

$$A_\rho = A_x \cos \phi + A_y \sin \phi = A_r \sin \theta + A_\theta \cos \theta$$

$$A_\phi = -A_x \sin \phi + A_y \cos \phi$$

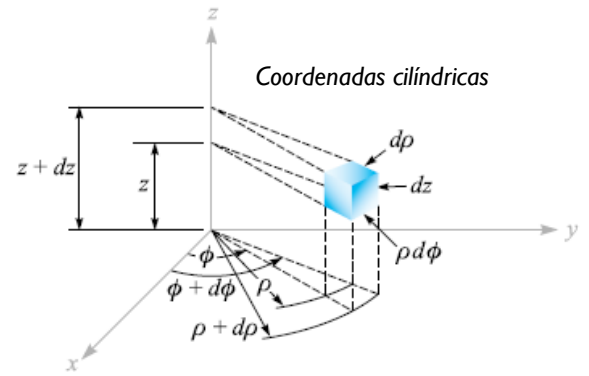
$$A_r = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta = A_\rho \sin \theta + A_z \cos \theta$$

$$A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta = A_\rho \cos \theta - A_z \sin \theta$$

Cálculo Vetorial

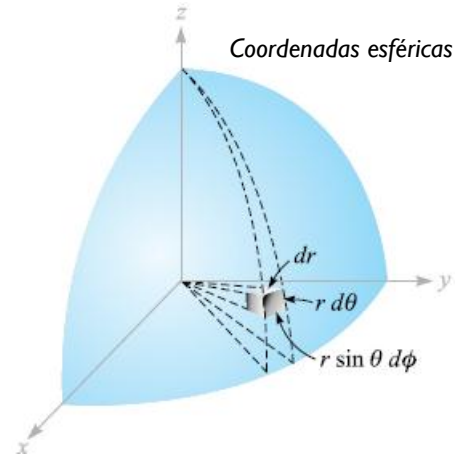
Elementos vetoriais diferenciais de comprimento

$$d\mathbf{L} = \begin{cases} \mathbf{a}_x dx + \mathbf{a}_y dy + \mathbf{a}_z dz \\ \mathbf{a}_\rho d\rho + \mathbf{a}_\phi \rho d\phi + \mathbf{a}_z dz \\ \mathbf{a}_r dr + \mathbf{a}_\theta r d\theta + \mathbf{a}_\phi r \sin \theta d\phi \end{cases}$$



Elementos vetoriais diferenciais de área

$$d\mathbf{S} = \begin{cases} \mathbf{a}_x dy dz + \mathbf{a}_y dx dz + \mathbf{a}_z dx dy \\ \mathbf{a}_\rho \rho d\phi dz + \mathbf{a}_\phi \rho dz + \mathbf{a}_z \rho d\rho d\phi \\ \mathbf{a}_r r^2 \sin \theta d\theta d\phi + \mathbf{a}_\theta r \sin \theta dr d\phi + \mathbf{a}_\phi r dr d\theta \end{cases}$$



Elementos diferenciais de volume

$$dV = \begin{cases} dx dy dz \\ \rho d\rho d\phi dz \\ r^2 \sin \theta dr d\theta d\phi \end{cases}$$

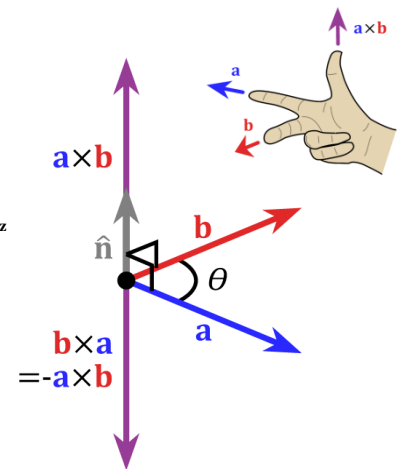
Operações multiplicativas vetoriais

(válidas para qualquer sistema de coordenadas)

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos(A \angle B) = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \sin(A \angle B) \mathbf{a}_n =$$

$$= (A_y B_z - A_z B_y) \mathbf{a}_x + (A_z B_x - A_x B_z) \mathbf{a}_y + (A_x B_y - A_y B_x) \mathbf{a}_z$$



Coordenadas retangulares –

$$\begin{cases} \mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z \\ \mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x \\ \mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y \end{cases}$$

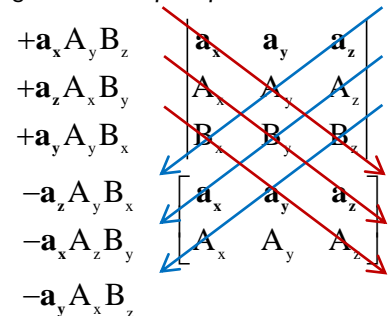
Coordenadas cilíndricas –

$$\begin{cases} \mathbf{a}_\rho \times \mathbf{a}_\phi = \mathbf{a}_z \\ \mathbf{a}_\phi \times \mathbf{a}_z = \mathbf{a}_\rho \\ \mathbf{a}_z \times \mathbf{a}_\rho = \mathbf{a}_\phi \end{cases}$$

Coordenadas esféricas –

$$\begin{cases} \mathbf{a}_r \times \mathbf{a}_\theta = \mathbf{a}_\phi \\ \mathbf{a}_\theta \times \mathbf{a}_\phi = \mathbf{a}_r \\ \mathbf{a}_\phi \times \mathbf{a}_r = \mathbf{a}_\theta \end{cases}$$

Regra de Sarrus para produto vetorial:



Cálculo Vetorial

Operadores vetoriais diferenciais

- $\nabla\psi$ → **gradiente** de um campo escalar ψ (retorna um campo vetorial)
 $\nabla \cdot \mathbf{A}$ → **divergente** de um campo vetorial \mathbf{A} (retorna um campo escalar)
 $\nabla \times \mathbf{A}$ → **rotacional** de um campo vetorial \mathbf{A} (retorna um campo vetorial)
 $\nabla^2\psi$ → **Laplaciano** de um campo escalar ψ (retorna um campo escalar)
 $\nabla^2\mathbf{A}$ → **Laplaciano vetorial** de um campo vetorial \mathbf{A} (retorna um campo vetorial)

Coordenadas retangulares:

$$\nabla\psi = \frac{\partial\psi}{\partial x}\mathbf{a}_x + \frac{\partial\psi}{\partial y}\mathbf{a}_y + \frac{\partial\psi}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \mathbf{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \mathbf{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \mathbf{a}_z$$

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \nabla \cdot \nabla\psi$$

$$\nabla^2\mathbf{A} = \nabla^2 A_x \mathbf{a}_x + \nabla^2 A_y \mathbf{a}_y + \nabla^2 A_z \mathbf{a}_z = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times (\nabla \times \mathbf{A})$$

Coordenadas cilíndricas:

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\mathbf{a}_\phi + \frac{\partial\psi}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z} \right) \mathbf{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho} \right) \mathbf{a}_\phi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi} \right) \mathbf{a}_z$$

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$$

$$\nabla^2\mathbf{A} = \left(\nabla^2 A_\rho - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial\phi} - \frac{A_\rho}{\rho^2} \right) \mathbf{a}_\rho + \left(\nabla^2 A_\phi - \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\phi} - \frac{A_\phi}{\rho^2} \right) \mathbf{a}_\phi + \nabla^2 A_z \mathbf{a}_z$$

Cálculo Vectorial

Coordenadas esféricas:

$$\begin{aligned}\nabla\psi &= \frac{\partial\psi}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\operatorname{sen}\theta}\frac{\partial\psi}{\partial\phi}\mathbf{a}_\phi \\ \nabla\cdot\mathbf{A} &= \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial(A_\theta\operatorname{sen}\theta)}{\partial\theta} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial A_\phi}{\partial\phi} \\ \nabla\times\mathbf{A} &= \frac{1}{r\operatorname{sen}\theta}\left(\frac{\partial(A_\phi\operatorname{sen}\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right)\mathbf{a}_r + \\ &\quad + \frac{1}{r}\left(\frac{1}{\operatorname{sen}\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right)\mathbf{a}_\theta + \\ &\quad + \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)\mathbf{a}_\phi \\ \nabla^2\psi &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2\psi}{\partial\phi^2} \\ \nabla^2\mathbf{A} &= \left[\nabla^2A_r - \frac{2A_r}{r^2} - \frac{2}{r^2\operatorname{sen}\theta}\left(\frac{\partial(A_\theta\operatorname{sen}\theta)}{\partial\theta} + \frac{\partial A_\phi}{\partial\phi}\right)\right]\mathbf{a}_r + \\ &\quad + \left[\nabla^2A_\theta - \frac{A_\theta}{r^2\operatorname{sen}^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cos\theta}{r^2\operatorname{sen}^2\theta}\frac{\partial A_\phi}{\partial\phi}\right]\mathbf{a}_\theta + \\ &\quad + \left[\nabla^2A_\phi - \frac{A_\phi}{r^2\operatorname{sen}^2\theta} + \frac{2}{r^2\operatorname{sen}\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cos\theta}{r^2\operatorname{sen}^2\theta}\frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_\phi\end{aligned}$$

Propiedades:

$$\begin{aligned}\nabla(\psi + \phi) &= \nabla\psi + \nabla\phi & \nabla\times\nabla\psi &= 0 \\ \nabla\cdot(\mathbf{A} + \mathbf{B}) &= \nabla\cdot\mathbf{A} + \nabla\cdot\mathbf{B} & \nabla\cdot(\nabla\times\mathbf{A}) &= 0 \\ \nabla\times(\mathbf{A} + \mathbf{B}) &= \nabla\times\mathbf{A} + \nabla\times\mathbf{B}\end{aligned}$$

Identidades:

$$\begin{aligned}\nabla\cdot\nabla\psi &= \nabla^2\psi \\ \nabla\times(\nabla\times\mathbf{A}) &= \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} \\ \nabla\cdot(\psi\mathbf{A}) &= [\mathbf{A}\cdot\nabla\psi] + [\psi\nabla\cdot\mathbf{A}] \\ \nabla\cdot(\mathbf{A}\times\mathbf{B}) &= (\nabla\times\mathbf{A})\cdot\mathbf{B} - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \\ \nabla\times(\psi\mathbf{A}) &= [\psi(\nabla\times\mathbf{A})] + [(\nabla\psi)\times\mathbf{A}] \\ \nabla^2(\phi\psi) &= \phi\nabla^2\psi + 2(\nabla\phi\cdot\nabla\psi) + \psi\nabla^2\phi\end{aligned}$$

Cálculo Vetorial

Teoremas de cálculo vetorial

Teorema da Divergência:

$$\oiint_S \mathbf{A} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{A} dV$$

Teorema de Stokes:

$$\oint_C \mathbf{A} \cdot d\mathbf{L} = \iint_S \nabla \times \mathbf{A} \cdot d\mathbf{S}$$

Circulação de um campo conservativo \mathbf{F} (campo irrotacional):

Se $\int_P \mathbf{F} \cdot d\mathbf{L}$ independe do caminho P , então $\oint_C \mathbf{F} \cdot d\mathbf{L} = 0$ e $\nabla \times \mathbf{F} = 0$.

Para $\oint_C \mathbf{F} \cdot d\mathbf{L} = 0$ e $\nabla \times \mathbf{F} = 0$, $\mathbf{F} = \nabla\psi$ (pois $\nabla \times \nabla\psi = 0$).

Álgebra complexa e fasores

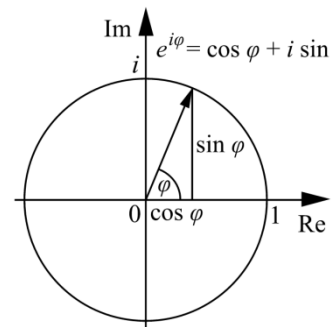
Unidade imaginária: $j^2 = -1$

Complexos conjugados:

$$(\alpha + j\beta)^* = (\alpha - j\beta)$$

$$(\alpha + j\beta)(\alpha + j\beta)^* = \alpha^2 + \beta^2$$

$$\frac{(\sigma + j\omega)}{(\alpha + j\beta)} = \frac{(\sigma\alpha + \omega\beta) - j(\sigma\beta - \omega\alpha)}{\alpha^2 + \beta^2}$$



Fórmula de Euler: $e^{j\varphi} = \cos \varphi + j \sin \varphi$

Forma polar (fasores):

$$z = \sigma + j\omega = Z e^{j\varphi}$$

$$\begin{cases} \sigma = Z \cos \varphi \\ \omega = Z \sin \varphi \\ Z = |z| = \sqrt{\sigma^2 + \omega^2} \\ \varphi = \tan^{-1} \left(\frac{\omega}{\sigma} \right) \end{cases}$$

se $\begin{cases} z = (\sigma + j\omega) = Z e^{j\varphi} \\ p = (\alpha + j\beta) = P e^{j\phi} \end{cases}$

$$z \cdot p = (\alpha + j\beta)(\sigma + j\omega) = M P e^{j(\theta+\phi)}$$

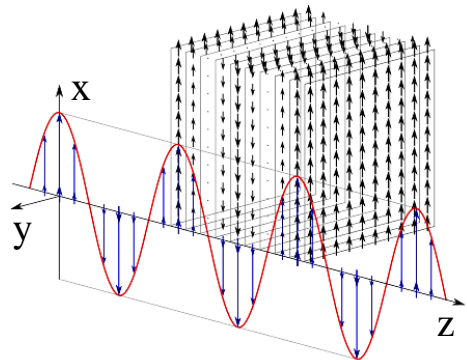
$$\frac{z}{p} = \frac{(\alpha + j\beta)}{(\sigma + j\omega)} = \frac{M}{P} e^{j(\theta-\phi)}$$

Equação de onda

Unidimensional: $\frac{\partial^2 u(z, t)}{\partial t^2} = c^2 \frac{\partial^2 u(z, t)}{\partial z^2}$

Geral: $\frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi = 0$

Solução: qualquer função do tipo $\psi = \psi(z \pm ct)$
(onda viajante em z com velocidade c)

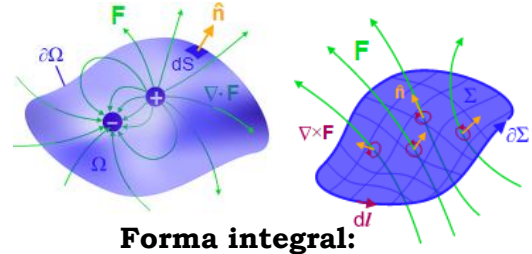


Eletromagnetismo

Relações constitutivas

Densidade de corrente de condução (lei de Ohm vetorial):	$\mathbf{J} = \sigma \mathbf{E}$	(A/m ²)
Densidade de fluxo elétrico (vetor deslocamento elétrico):	$\mathbf{D} = \epsilon \mathbf{E}$	(C/m ²)
Densidade de fluxo magnético (vetor indução magnética):	$\mathbf{B} = \mu \mathbf{H}$	(T = Wb/m ²)

Equações de Maxwell



	Forma diferencial (pontual):	Forma integral:
Lei de Gauss: (cargas elétricas geram campos elétricos)	$\nabla \cdot \mathbf{D} = \rho_v$	$\oiint_{S=\partial\Omega} \mathbf{D} \cdot d\mathbf{S} = Q$
Lei de Gauss para o magnetismo: (não existem monopolos magnéticos)	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{S=\partial\Omega} \mathbf{B} \cdot d\mathbf{S} = 0$
Lei de Faraday-Maxwell: (campos magnéticos variantes no tempo induzem campos elétricos)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{C=\partial\Sigma} \mathbf{E} \cdot d\mathbf{L} = -\frac{\partial}{\partial t} \iint_{\Sigma} \mathbf{B} \cdot d\mathbf{S}$
Lei de Ampère-Maxwell: (campos magnéticos são gerados por corrente elétrica e campos elétricos variantes no tempo)	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_{C=\partial\Sigma} \mathbf{H} \cdot d\mathbf{L} = I + \frac{\partial}{\partial t} \iint_{\Sigma} \mathbf{D} \cdot d\mathbf{S}$

nas quais Ω é um volume fechado de fronteira $S = \partial\Omega$ (superfície fechada)

e Σ é uma superfície aberta de fronteira $C = \partial\Sigma$ (curva fechada).

- Carga elétrica total contida em um volume Ω : $Q = \iiint_{\Omega} \rho_v dV$
- Corrente elétrica de condução através de uma superfície Σ : $I = \iint_{\Sigma} \mathbf{J} \cdot d\mathbf{S}$

“Campos elétricos e magnéticos variantes no tempo que não satisfazem a estas equações não podem existir. Eletricidade, magnetismo e luz são todas manifestações dum mesmo fenômeno – o fenômeno eletromagnético.”

Princípio da conservação de carga

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho_v}{\partial t} = 0 \quad (\text{eq. continuidade, LKC vetorial})$$

Constantes no vácuo:

$$\epsilon_0 = 8,8541878176 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Eletrostática

Lei de Coulomb

$$\mathbf{E} = \frac{q}{4\pi\epsilon} \frac{(\mathbf{r}_m - \mathbf{r}_q)}{|\mathbf{r}_m - \mathbf{r}_q|^3} \quad (\text{V/m})$$

Fluxo elétrico e Lei de Gauss

$$\Psi = \oiint_S \mathbf{D} \cdot d\mathbf{S} = Q \quad (\text{C})$$

(1ª eq. Maxwell)

Energia e Potencial Elétrico

$$W_{AB} = qV_{AB} \quad (\text{J})$$

$$ddp = V_{AB} = -\int_B^A \mathbf{E} \cdot d\mathbf{L} \quad (\text{V} = \text{J/C})$$

Como $\oint_C \mathbf{E} \cdot d\mathbf{L} = 0$ (campo conservativo, LKT vetorial),

então existe um campo escalar V tal que

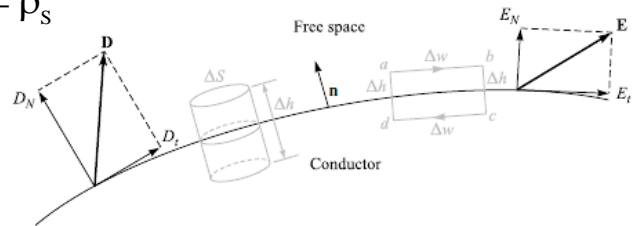
$$\mathbf{E} = -\nabla V$$

(consequência da 3ª eq. Maxwell para $\frac{\partial \mathbf{B}}{\partial t} = 0$)

Condições de fronteira

$$\text{Condutor-espaço livre - } \begin{cases} D_t = E_t = 0 \\ D_n = \epsilon_0 E_n = \rho_s \end{cases}$$

$$\text{Dielétrico-dielétrico - } \begin{cases} E_{t1} = E_{t2} \\ D_{n1} = D_{n2} \end{cases}$$



Capacitância

$$C = \frac{Q}{V_{AB}} = \frac{\oiint_S \mathbf{D} \cdot d\mathbf{S}}{-\int_{B(-)}^{A(+)} \mathbf{E} \cdot d\mathbf{L}} = \text{cte.} \quad (\text{F} = \text{C/V})$$

Equações de Poisson & Laplace

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

$$\nabla^2 V = 0$$

Magnetostática

Lei de Biot-Savart

$$d\mathbf{H} = \frac{I d\mathbf{L} \times (\mathbf{r}_m - \mathbf{r}_I)}{4\pi |\mathbf{r}_m - \mathbf{r}_I|^3}$$

Superposição:
$$\mathbf{H} = \int_C \frac{I d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} \quad (\text{A/m})$$

Lei Circuital de Ampère

$$\oint_C \mathbf{H} \cdot d\mathbf{L} = I$$

(4ª eq. Maxwell para $\frac{\partial \mathbf{D}}{\partial t} = 0$)

Fluxo magnético e Lei de Gauss para o magnetismo

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{S} \quad (\text{Wb} = \text{V} \cdot \text{s})$$

$$\oiint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

(2ª eq. Maxwell)

Potencial magnético vetorial

$$\mathbf{A} = \int_C \frac{\mu I d\mathbf{L}}{4\pi R} \quad (\text{V} \cdot \text{s/m})$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\Phi = \oint_C \mathbf{A} \cdot d\mathbf{L}$$

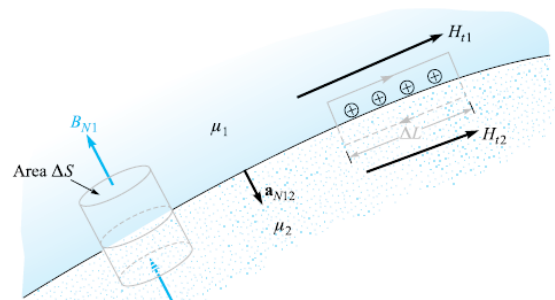
(consequência da 2ª eq. Maxwell)

Condições de fronteira

Entre dois meios -
$$\begin{cases} B_{n1} = B_{n2} \\ H_{t1} - H_{t2} = K_n \end{cases}$$

Indutância

$$L = \frac{\Phi}{I} = \frac{\iint_S \mathbf{B} \cdot d\mathbf{S}}{\oint_C \mathbf{H} \cdot d\mathbf{L}} = \text{cte.} \quad (\text{H} = \text{Wb/A})$$



Equação vetorial de Poisson

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

Eletrodinâmica Clássica

Força de Lorentz

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (\text{N})$$

Lei de Faraday-Lenz

$$\text{f.e.m.} = -N \frac{d}{dt} \Phi \quad (\text{V})$$

Equações de Maxwell no vácuo

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

Acoplamento fundamental
entre os campos
magnético e elétrico:

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$
$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Equações da onda eletromagnética

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\mu_0 \varepsilon_0} \nabla^2 \mathbf{E} = 0$$

$$\frac{\partial^2 \mathbf{H}}{\partial t^2} - \frac{1}{\mu_0 \varepsilon_0} \nabla^2 \mathbf{H} = 0$$

Em 3D: $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \mathbf{E}(x, y, z, t) = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}(x, y, z, t)}{\partial t^2}$

Onda eletromagnética plana no espaço livre

(campos ctes. em x e y, $k = \omega \sqrt{\mu_0 \varepsilon_0} = 2\pi/\lambda$ e $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, propaga em z com $c_0 = \omega/k = \sqrt{1/\mu_0 \varepsilon_0}$)

$$\mathbf{E}(z, t) = E_{x0} e^{j(\omega t - kz)} \mathbf{a}_x = E_x(z, t) \mathbf{a}_x$$

Como $\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} = \frac{\partial E_x}{\partial z} \mathbf{a}_y$,

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \times 10^8 \text{ m/s}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 c_0 \approx 120\pi \Omega$$

$$\mathbf{H}(z, t) = \frac{1}{\eta_0} E_{x0} e^{j(\omega t - kz)} \mathbf{a}_y = \frac{1}{\eta_0} E_x(z, t) \mathbf{a}_y = H_y(z, t) \mathbf{a}_y$$

Vetor de Poynting

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} \quad (\text{W/m}^2)$$

$$P_{\text{med}} = \frac{1}{T} \int_0^T \left[\iint_S \mathbf{S} \cdot d\mathbf{S} \right] dt$$

Efeitos Eletromagnéticos em Alta Frequência

Efeito pelicular

$$\mathbf{J}(r) = \mathbf{J}_s e^{-r/\delta}$$

Profundidade de penetração -
$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

Linhas de transmissão

Equações do telégrafo (p/ correntes e tensões senoidais, LT ao longo de z):

$$\begin{cases} \frac{d}{dz} V(z, t) = -R I(z, t) - L \frac{d}{dt} I(z, t) \\ \frac{d}{dz} I(z, t) = -G V(z, t) - C \frac{d}{dt} V(z, t) \end{cases} = \begin{cases} \frac{\partial^2}{\partial z^2} V(z) - \gamma^2 V(z) = 0 \\ \frac{\partial^2}{\partial z^2} I(z, t) - \gamma^2 I(z) = 0 \end{cases}$$

Solução - $V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$

Cte. propagação - $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \cong j\omega\sqrt{LC}$

Impedância Característica - $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \frac{R + j\omega L}{\gamma} \cong \sqrt{\frac{L}{C}}$

Velocidade de fase - $v_p = \frac{\omega}{\beta} \cong \frac{1}{\sqrt{LC}}$

Coefficiente de reflexão - $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

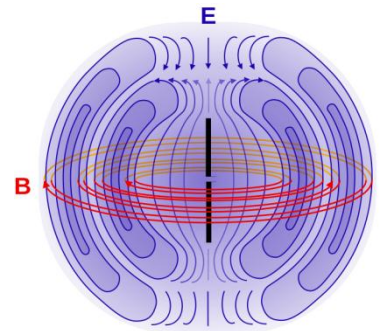


L (H/m), C(F/m),
R(Ω/m), G (S/m)

Antenas

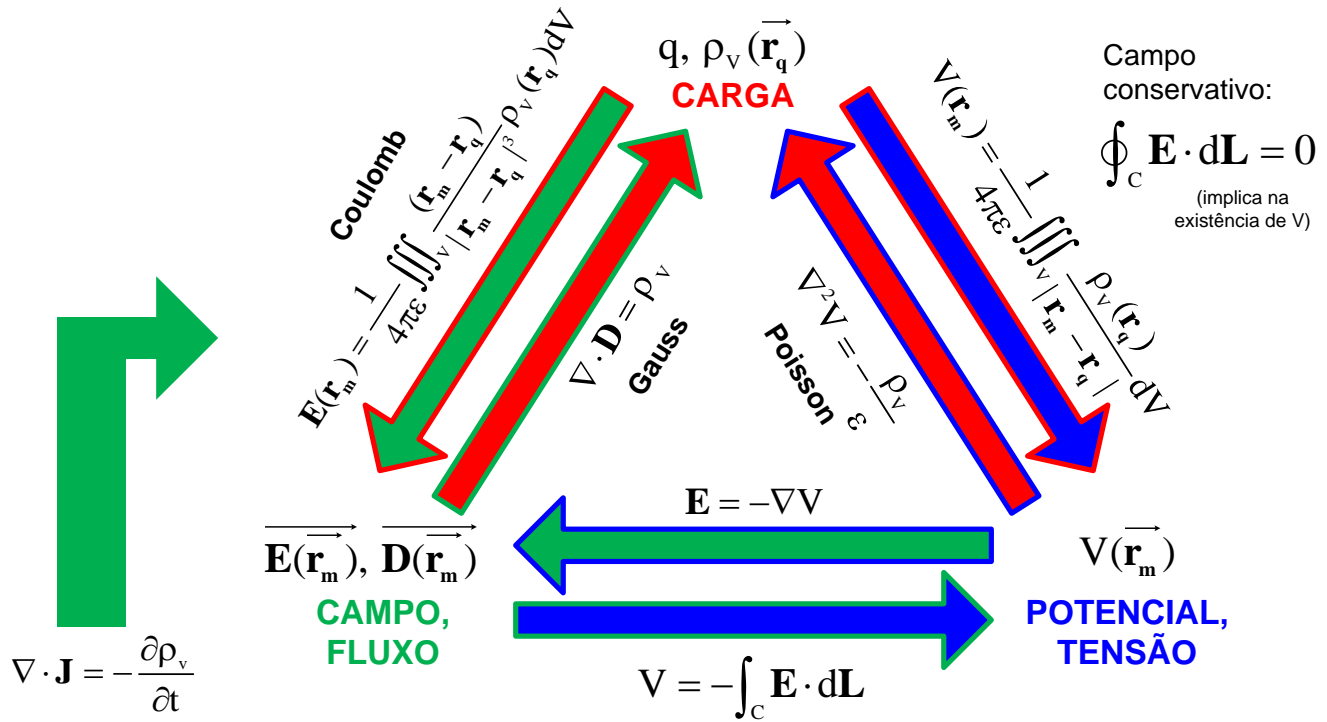
Dipolo Hertziano ($d \ll \lambda$) - $\mathbf{A} = \frac{\mu I d}{4\pi R} \mathbf{a}_z$

Solução para campo distante -
$$\begin{cases} H_\phi = j \frac{I d k^2}{4\pi k r} e^{-jkr} \sin \theta \\ E_\theta = j \frac{I d k^2}{4\pi k r} \sqrt{\frac{\mu_0}{\epsilon_0}} e^{-jkr} \sin \theta \end{cases}$$



Resistência de radiação - $R_{rad} = 2 \frac{P_{med}}{I^2} = \eta_0 \frac{2\pi}{3} \frac{d^2}{\lambda^2}$

Triângulo de Leis da Eletrostática em \mathbb{R}^3



$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$

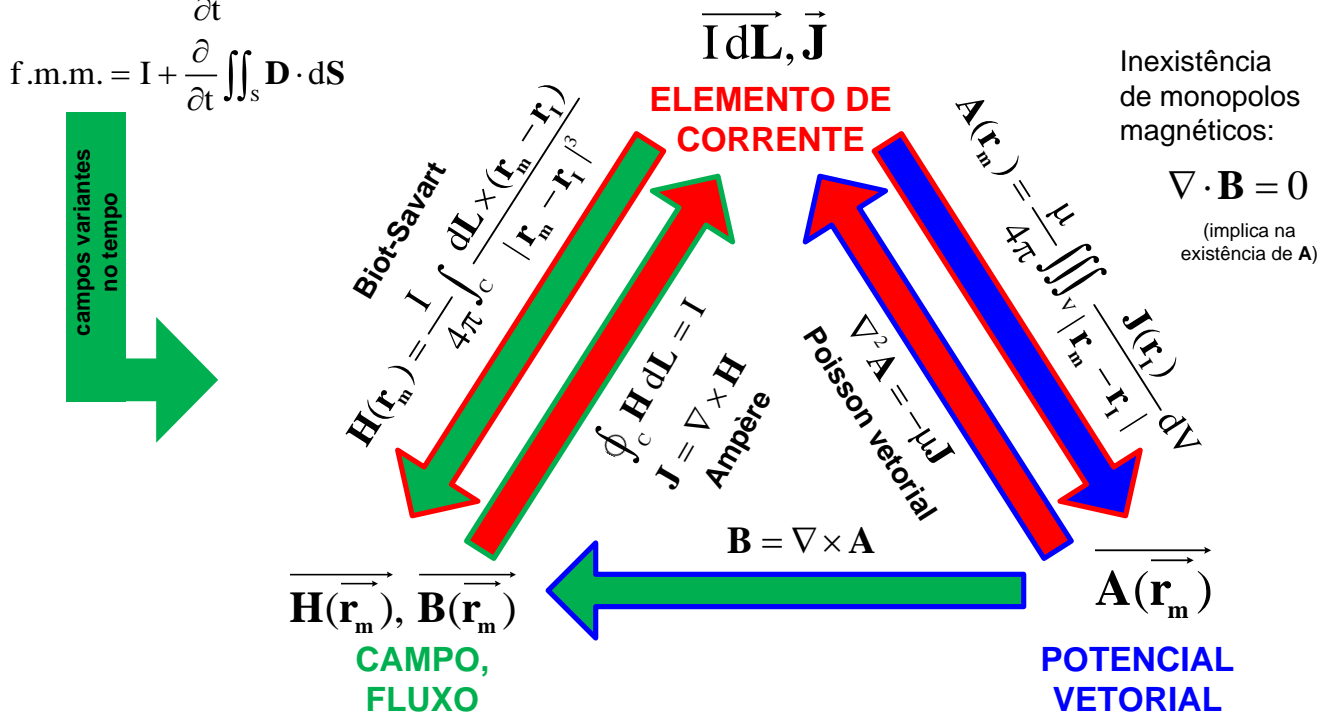
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

f.e.m. = $-\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{S}$

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

f.m.m. = $I + \frac{\partial}{\partial t} \iint_S \vec{D} \cdot d\vec{S}$

Triângulo de Leis da Magnetostática em \mathbb{R}^3



campos variantes no tempo