

AN EXPERIMENTAL STUDY ON THE PHOTOMETRICAL IMPACTS OF SEVERAL CURRENT WAVEFORMS ON POWER WHITE LEDs

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Abstract – Since light-emitting diodes (LEDs) are a rising lighting technology, a great amount of research is currently being focused on driving techniques for these current-fed semiconductor devices. For realizing efficient current drivers, power electronics is usually employed. This work focuses on studying the impacts that several high and low frequency current waveforms commonly found on power electronic drivers may have on the photometrical and colorimetical performance of the LEDs themselves. Empirical evidence obtained under controlled experiments is provided to correlate important features of the LEDs to the current levels and waveforms.

Keywords – Colorimetry, Current Waveform, Light-Emitting Diode Drivers, Photometry, Power White LEDs, Solid-State Lighting

I. INTRODUCTION

Light-emitting diodes (LEDs) are a promising lighting alternative under several perspectives. Some estimations are that energy consumption for producing artificial lighting could be reduced up to 50% [1] using them as the “green” substitute of current filament or gas discharge-based lamps. High luminous efficiency, high color rendering index (CRI), long lifespan and white light emission makes them good candidates for applications on general indoor lighting, as well as being verifiably suitable for use in public illumination, under the typical low mesopic conditions of street lighting [2].

Power LEDs are a special case within the high-brightness LED (HB-LED) family, operating at higher currents, some up to 1000 mA [3], making them more suitable for building luminaires by reducing the required number of devices employed on a single unit to provide adequate luminous flux.

LEDs are essentially current driven devices [4]-[6], since they are unable to regulate their own current if supplied by voltage sources. Therefore, LED drivers are the electronic circuits which intend to control their forward current.

For such high current power LEDs, power electronics is usually chosen for drivers and current control, due to some implications of high frequency switch-mode operation: higher efficiency and reduced volume of the circuit. The need for power factor correction (PFC) on the drivers supplied by mains also points to two-stage [7], single-stage [8] or integrated switching power converters [9], [10].

Switching solutions do not produce a pure DC current waveform on their output, though. Quasi-DC low-ripple waveforms can be obtained at the expense of using bulky output filters, increasing cost and volume, or by using higher switching frequencies, but not without increasing also the

switching losses and aggravating EMI-related issues. Also, dimming the LEDs under pulse width-modulated (PWM) current exposes the device to repetitive current pulses.

The impacts of rippled or pulsed high frequency currents on the photometrical and colorimetical performance of LEDs have been assessed in some past works [11]-[15] and some research has been focused on reducing the peak-to-average ratio of LED current [4], [8], [16].

Also, some low-cost LED drivers have been proposed to work on low frequency, fed by mains and using a rectifier or the LEDs themselves as rectifiers, together with a limiting combination of passive components (e.g., a capacitor, inductor or a resistor) [17]. Indeed, some of these solutions can be readily found as somewhat cheap LED lamps on the market. The impacts of these current waveforms on LED photometrical performance and possible shortening of lifespan are not yet clearly known.

This paper presents the outcome of an experimental study regarding the impacts on flux, luminous efficiency, color shift and spectral deviation on LEDs under several current waveforms common to switch-mode circuits (high frequency quasi-square and triangular waves) and as well to low frequency rectified sinusoidal currents (full and half-wave).

Tests conducted under pure DC (stiff constant current) are provided as reference, as well as a comparison between different current waveforms of the same average value, tested on a controlled-temperature experiment. Conclusions can be drawn from the evidences in order to improve or establish design guidelines for power electronic drivers that ensure maintenance of LED performance as a light source.

II. GENERAL CHARACTERISTICS OF POWER PHOSPHOR-CONVERTED WHITE LEDs

Most of the currently commercially available white LEDs are made of nitride-based semiconductors (InGaN, GaN) [12], [19], which emit blue light (ca. 440 nm wavelength), coated with a cerium-doped yttrium-aluminum garnet (YAG) phosphor. This scheme is to shift part of the short wavelength blue light to longer wavelengths through phosphorescence, producing white light altogether, with a very characteristic continuous emission spectrum.

Lifespan of these nitride-based LEDs has been estimated to be in excess of 40,000 hours under accelerated lifetime tests and severe thermal stress [18][17].

Being a P-N junction, LEDs can be electrically modeled by a modified Shockley equation [19], such as (1).

$$V_{LED}(I_{LED}) = \frac{nkT}{q} \cdot \ln\left(\frac{I_{LED}}{I_S}\right) + R_S I_{LED} \quad (1)$$

Where:

- V_{LED} - LED voltage drop.
- I_{LED} - LED forward current.
- n - Ideality factor.
- k - Boltzmann constant ($1.3806504 \times 10^{-23} \text{ J.K}^{-1}$).
- T - Room temperature, in Kelvins (K).
- q - Elementary charge ($1.602176487 \times 10^{-19} \text{ C}$).
- I_S - Reverse bias saturation current, in Amperes.
- R_S - Parasitic series resistance of the LED.

Equation (1) expresses LED voltage as the sum of a linear and a logarithmic function of the current. This is physically represented by a real diode in series with a resistance R_S .

By testing an Osram LUW W5PM Golden Dragon Plus white LED (106 lm @ 350 mA) [3] under DC and constant temperature ($T = 28^\circ \text{C}$, or 301 K), it was possible to derive the parameters compiled in Table I for this particular device.

TABLE I
Osram LUW W5PM parameters, T = 301 K

Parameter	Value
$\frac{nkT}{q}$	165 mV
I_S	2.3 nA
R_S	412 m Ω

The V-I characteristic of this LED matches the experimental points obtained on the DC test, as shown in Fig. 1. The relative spectral power distribution (SPD) for this LED has also been obtained with the use of an integrating sphere and spectrophotometer, and is shown in Fig. 2, normalized to the peak radiant flux.

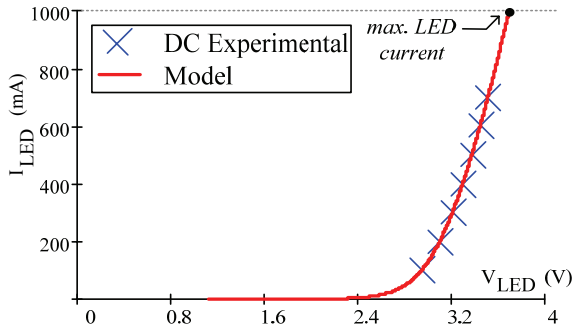


Fig. 1. V-I characteristic of the LED used on this experiment.

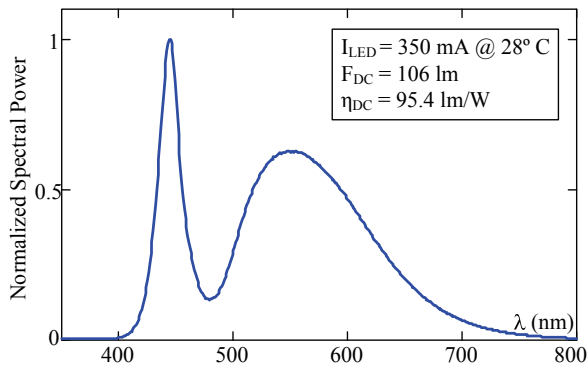


Fig. 2. Relative spectral power distribution of tested LED.

Other two interesting relations obtained from the reference DC test with the LUW W5PM white LED are depicted in Fig. 3. It shows how luminous flux (normalized to the measured value of $F_{DC} = 106 \text{ lm}$, $\eta_{DC} = 95.4 \text{ lm/W}$, at $I_{LED} = 350 \text{ mA}$) tends to saturate with increasing DC currents, hence the drop in luminous efficiency also shown, on the secondary axis. This behavior, called, “droop”, is unrelated to temperature [20]. Flux measurements were taken on an integrating sphere for each of the values of current shown in Fig. 1. Then a 4th order polynomial regression was used to obtain an approximation curve for the normalized flux, and an exponential regression was used for the efficiency curve.

Parallel experiments, [14]-[15], though using illuminance rather than luminous flux, show similar results; so does [11] (for luminous flux).

These three curves conclude the general characterization of the studied LED, tested under DC bias and controlled temperature. These characteristics should be similar to other phosphor converter GaN/InGaN-based white LEDs. The DC test is to be used as the reference (control) experiment for further tests presented in this work.

III. LED PERFORMANCE UNDER HIGH FREQUENCY CURRENT WAVEFORMS

For the high frequency experiment with the LUW W5PM LED, four positive-only current waveforms were chosen: a 35% and a 70% duty-cycle quasi-square wave (QSW) for representing pulse width modulation (PWM) dimming conditions and two quasi-triangular currents, for representing output inductor rippled-conditions of common current-output LED drivers (e.g., a buck converter). For these triangle waves, 20% and 200% ripples were tested, as if in a converter working under continuous conduction mode (CCM) for the first case and in critical conduction mode (CRM) for the second case.

Frequency was chosen 50 kHz, since it is common to have LED drivers operating at this frequency level [5], and evidence suggests that LEDs do not show photometric differences for current frequencies up to 400 kHz [11] or even as high as 2 MHz [15].

All waveforms have the same current average value of 350 mA. The quasi-triangular current waveforms were synthesized by using a buck converter with variable output inductor, while the quasi-square waves were generated by switching a non-ideal current source through the LED. The experimentally obtained waveforms are shown in Fig. 4.

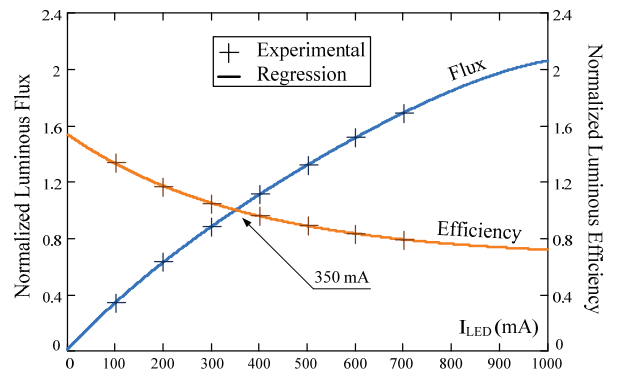


Fig. 3. Normalized flux and luminous efficiency versus current.

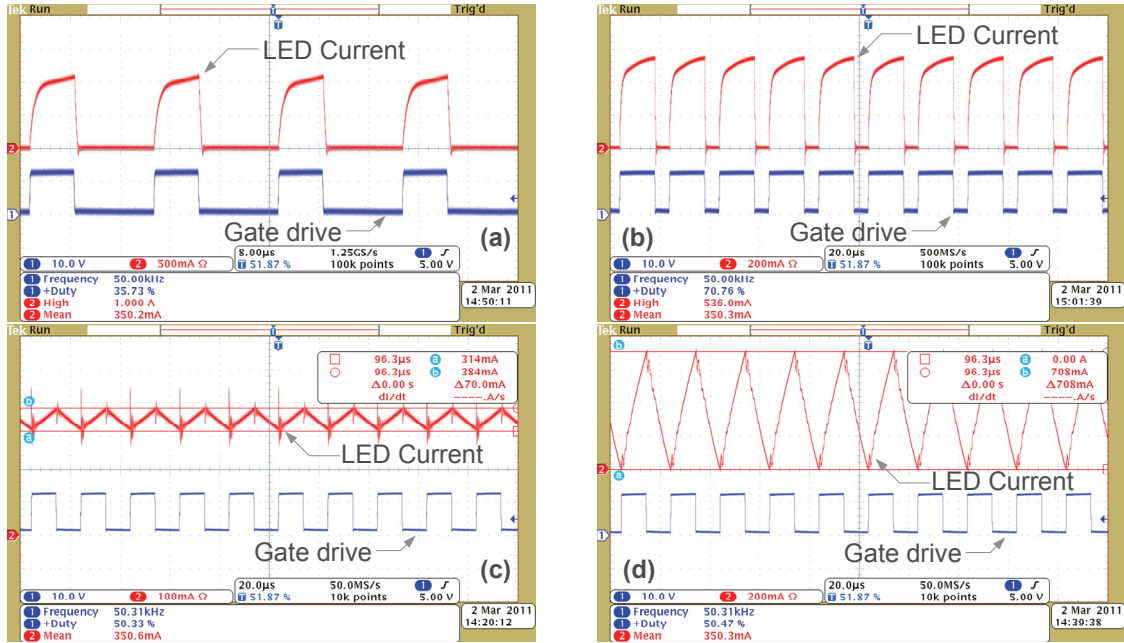


Fig. 4. Waveforms for the high frequency experiment: (a) 35% QSW (8 μ s/div, 500 mA/div), (b) 70% QSW (20 μ s/div, 200 mA/div), (c) 20%-rippled CCM quasi-triangular (20 μ s/div, 100 mA/div) and (d) 200%-rippled CRM quasi-triangular (20 μ s/div, 200 mA/div).

Photometrical measurements were carried out in an integrating sphere for each experiment, in order to correlate the waveforms of Fig. 4 to possibly relevant data. The LED was mounted on a fixed-temperature heat sink (28° C).

Figures Fig. 5 and Fig. 6 show the spectral power distributions (SPD) for this LED on the presented tests.

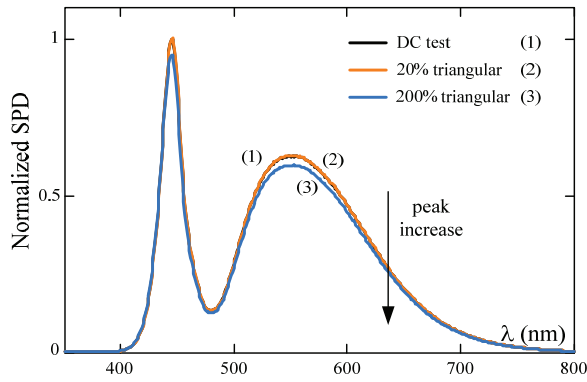


Fig. 5. Relative SPD for the DC and triangular waveforms, normalized to the DC peak radiant flux. Percentages denote ripple.

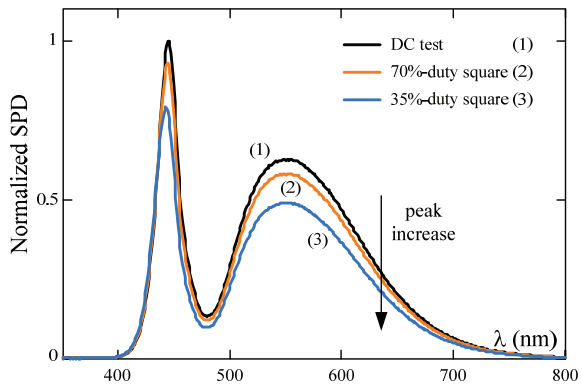


Fig. 6. Relative SPD for the DC and quasi-square waveforms, normalized to the DC peak radiant flux. Percentages denote duty-cycle of the square current.

The SPDs of all measurements were normalized and plotted together to the DC curve, to enable easy comparison between the current waveforms driving the LED.

In Fig. 5, it can be seen that the SPDs of DC and 20%-rippled triangular current are superimposed. This implies that virtually no photometrical difference can be detected between these two waveforms. As for the 200%-rippled triangular waveform, slight decrease in light power emission can be noticed.

In Fig. 6, though, great difference can be noticed between the two different duty-cycle QSW and the DC curve. Since 35% duty implies greater peak current than a 70% duty, flux decrease seems to be directly correlated to this peak in the current necessary to maintain the average value of both waveforms ($I_{LED} = 350$ mA).

The general decrease on the power distribution signals a decrease in luminous efficiency, since all SPDs were obtained under same average LED power.

Not only significant drops in efficiency, but also chromaticity shifts are reported when pulsed LED current is used for dimming or control [12], [13]. Some aspects relating the peak current of all experiments to color shifts will also be analyzed in a further section.

IV. LED PERFORMANCE UNDER LOW FREQUENCY CURRENT WAVEFORMS

As cited before, low frequency current waveforms could also be used to drive LEDs from mains. Some of the impacts on LED photometrical and colorimetical performances of these waveforms were assessed in the following experiment.

Two rectifier circuits, each with a limiting power resistor, were used to drive the LED from a variable sinusoidal source: a half-wave and a full-wave bridge rectifier. The current was also set at the average value of 350 mA, for both.

The waveforms of these two tests are shown in Fig. 7.

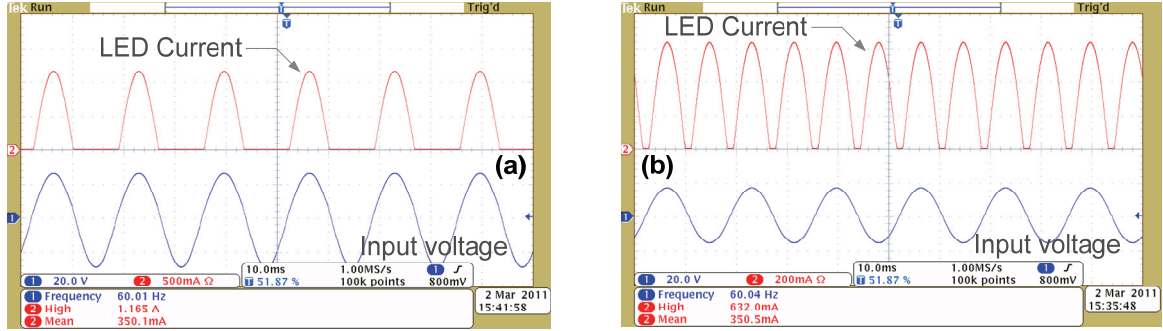


Fig. 7. Waveforms for the low frequency experiment: (a) half-wave rectified (10 ms/div, 500 mA/div) and (b) full-wave rectified (10 ms/div, 200 mA/div), from 60 Hz sinusoidal source. Turn-on delay due to LED threshold voltage can be clearly observed in (b).

It can be clearly noticed that a half-wave current needs a higher peak in order to maintain the same average value of a full-wave. This peak on the half-wave is almost twice as high as the full-wave current peak.

Once more, the SPDs for both experiments were obtained, at 28° C (heat sink), normalized to the DC peak radiant flux and plotted together. These are shown in Fig. 8.

Similar behavior correlating higher peaks to greater decreases in efficiency can be easily seen in this plot. The half-wave, having the higher current peak, is also the greatest drop on the general spectral distribution, and thus in efficiency, among both tested waveforms.

Slight blue shift on the peak wavelength can also be noticed, and this will be further analyzed together with the high frequency waveform tests.

It is important to stress that, though these two last tests were conducted at mains frequency (60 Hz), similar waveforms from high frequency converters (e.g., resonant converters) can be obtained [16]. Due to the frequency characteristic of the LEDs [11], [15], it is believed that the performance would be quite the same; up until a limiting cut-off frequency, LED photometrical performance should be only function of the waveform itself.

V. COMPARISON BETWEEN WAVEFORM INFLUENCES ON PHOTOMETRICS AND COLORIMETRICS

Taking into account all six conditions under which the LED was tested (four high frequency and two low frequency) plus the DC reference test, some interesting photometrical and colorimetical data can be compiled for comparison.

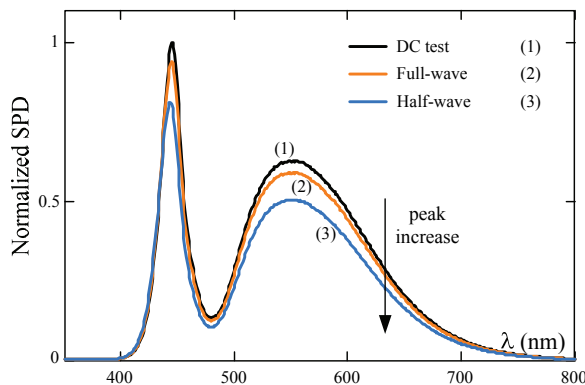


Fig. 8. Relative SPD for the DC and both rectified sine waveforms, normalized to the DC peak radiant flux.

A. Photometrical Comparison

The first aspect that can be analyzed from the tests is a comparison between the overall photometric performance of the LED under several waveforms and the DC reference test. The SPDs of Figures Fig. 5, Fig. 6 and Fig. 8 show qualitatively that some discrepancies rise when using various waveforms, but quantifying the relative luminous efficiency is a more useful comparison figure. Table II shows such comparison. All luminous efficiency values were normalized to the DC-obtained value (95.4 lm/W @ $I_{LED} = 350$ mA and $T = 28^\circ\text{C}$), thus making it the 100% reference test.

Peak current of the waveform was also compiled, in order to verify that higher peaks tend to decrease the luminous efficiency. But the table shows that the influence on photometrical performance is actually a conjugate of this peak and the waveform itself. For example, even though a 35% duty cycle quasi-square wave (QSW) current has a slightly lower peak current than the half-wave rectified sine one, the luminous efficiency is actually lower for the first, due to its inherent pulsed nature. It appears that since the half-wave has a “smoother” current rise (lower di/dt), an enhanced efficiency results, despite higher current peak.

Same behavior can be verified when comparing the 200%-rippled triangle wave or the full-wave rectified current to the 70% duty cycle QSW: higher peak for the first two waves, but greater relative luminous efficiency on both, probably due to the smoother behavior of the current rise.

B. Colorimetical Comparison

Color changes in LEDs were reported in some past works [12], [13], especially regarding the impact that dimming techniques (such as PWM or DC level) may have on the chromaticity coordinates. For this experiment, chromaticity shifts were investigated for all tested waveforms.

TABLE II
Luminous Efficiency Comparison at $I_{LED} = 350$ mA

Current Waveform	Measured Peak Current	Normalized Luminous Efficiency
Pure DC (reference test)	350 mA	100.0 %
20%-rippled 50 kHz triangular	384 mA	99.5 %
200%-rippled 50 kHz triangular	708 mA	95.1 %
70%-duty 50 kHz quasi-square	536 mA	93.0 %
35%-duty 50 kHz quasi-square	1000 mA	79.2 %
Full-wave rectified 60 Hz sine	632 mA	93.9 %
Half-wave rectified 60 Hz sine	1165 mA	81.1 %

At first, the chromaticity coordinates x and y from the CIE 1931 color space were obtained for each experiment with the help of the spectrophotometer of the integrating sphere. These coordinates were plotted on a plane, for all experimented waveforms, and are shown in Fig. 9.

It can be observed that there is a tendency of light color blue shift when increasing peak currents are used.

Comparing the DC coordinates to the ones from the 20%-rippled triangle wave, one can show that virtually no chromatic difference exists between such low rippled current and a pure DC current.

But when analyzing the 200%-rippled triangle wave, it turns out that, even though it has a relatively high current peak (twice the average value of current), the chromaticity shift was the lowest from all waveforms, apart from the 20%-rippled triangle wave. This provides good evidence that triangular current waveforms provoke little color change on the LED, even when severe ripple is present, favoring this particular waveform. This information had not yet been mentioned in reference works and may be of some relevance, given that triangular currents are typical on power electronic LED drivers which rely on output inductors as filters to reduce the peak-to-average ratio of current, such as in [8].

On the other hand, the extremely pulsed waveforms, such as the half-wave rectified sine and the 35% duty cycle square, presented the worst chromaticity shifts.

VI. COLORIMETRICAL BEHAVIOR UNDER PWM AND DC DIMMING CONDITIONS

Pulsed currents have been shown to yield comparatively less luminous efficiency than less rippled or non-pulsed waveforms in Section V. But since PWM is commonly used for LED dimming, some studies from references [12] and [13] tried to conclude whether chromaticity shifts are more aggravated under PWM dimming or under DC (or DC-like) dimming.

Thus, an experiment dimming the LED under these two driving conditions was performed to check for color change.

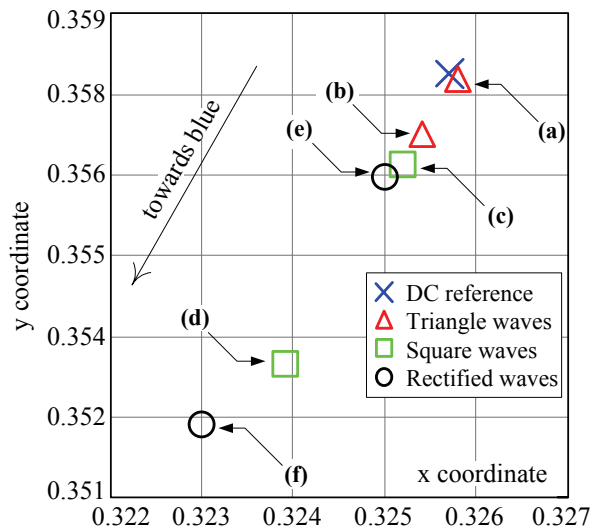


Fig. 9. Chromaticity coordinates of all experiments: (a) 20%-rippled triangular, (b) 200%-rippled triangular, (c) 70%-duty square, (d) 35%-duty square, (e) full-wave rectified sine, (f) half-wave rectified sine. DC coordinates are also indicated.

Fig. 10 shows the outcome of such test, experimenting with dimming levels from 100 mA to 800 mA on the same LUW W5PM LED, also using a 28° C constant temperature heat sink. Correlated color temperature (CCT) was used as the figure of merit of choice, because it can fairly represent the overall color characteristic of a white LED.

From Fig. 10, it is possible to conclude that both of the dimming methods imply in blue shift for increasing average current levels. Not only that, but also that, even though LEDs behave more inefficiently under squared waveforms, the color shift towards bluish-white light is actually less accentuated under PWM than under DC dimming. Thus, under the color stability aspect, PWM dimming is a more adequate method to conserve chromatic characteristic.

The assessment of chromaticity shift made in [13] conflicts with this evidence. It shows that PWM dimming implies in red shift, i.e., increasing current levels imply in a decrease in the CCT. But there is no mention of the LED model used or its general characteristics. In laboratory, similar results to this work (decrease in CCT) were obtained only when experimenting with an older LED model¹.

Color stability has been shown to be better under PWM dimming by Fig. 10, but [12] suggests that DC dimming offers better stability. Although a different LED was used in this work, the inference from [12] was made based upon a test which correlated the average forward current to the chromaticity coordinate x only, which does not represent the whole picture of the chromaticity shift alone. Several combinations of x and y coordinates could still result in a same CCT, thus same white color sensation. For white sources, it is perhaps more appropriate to use the figure of the CCT, rather than only the chromaticity coordinate x , to analyze the change in color of the LED as a whole, as done in this section of this paper.

VII. CONCLUSION

This paper presented an experimental approach to analyze some impacts that power electronic driving methods may have on the performance of PC white LEDs as a light source.

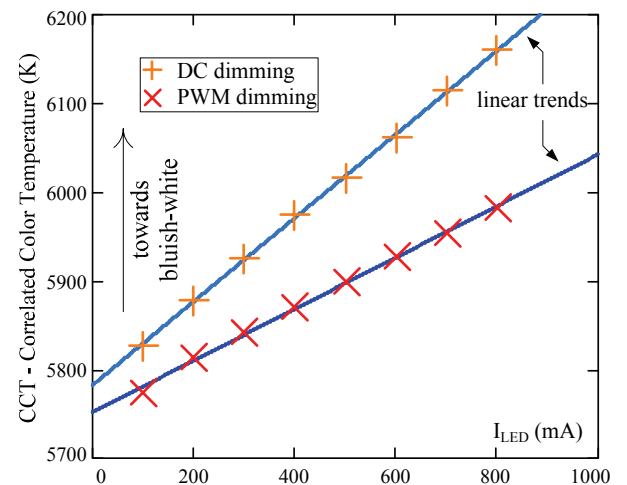


Fig. 10. Color temperature shifts under different dimming conditions. I_{LED} is the average value of LED current.

¹ A SunLED XZCW10X106W LED, 40 lm/W @ 700 mA, showed decreasing CCT (7471 K-6727 K) for increasing current (100 mA-700 mA).

Waveforms investigated are typical to power electronic LED drivers, such as DC-DC converters or AC-DC converters. Tests were carried out with a modern PC-LED model, mounted on a controlled-temperature heat sink, with data being gathered with the aid of a calibrated spectrophotometer and an integrating sphere.

Data analysis showed that smoother sloped waveforms in LED current generally imply in better light efficiency, such as in triangular waveforms. Lower ripples are correlated to better photometrical performance, but even largely rippled currents which have a smoother slope provided better efficiency than less rippled (or less peaked) currents that were, though, more pulsed.

The slight difference in luminous efficiency between 20%-rippled and 200%-rippled triangle waves proved that a large ripple could be handled by the LED under this waveform without great detriments to the photometrical and colorimetical performances. This implies that great reduction in output inductor filters could be made, decreasing cost, volume and copper losses in the magnetic elements of current source-type converters which rely on this scheme.

The performance under low frequency rectified sine waves was verified as being fairly good under the full-wave rectified current, but severely impaired under the half-wave current. Also, severe flickering resulted from this last, since the LED remained out during half cycle of the 60 Hz current, enough to be noticed by the human eye. Thus, though full-wave currents could be used under high or low frequency, the employment of half-wave currents in LED drivers is greatly discouraged, and almost unviable for low frequency.

The impact of dimming techniques on the color stability of the PC-LED was also investigated. It was shown that blue shift is a main characteristic of increasing the average current on LEDs, but also that using a PWM technique for such task results in less color deviation towards bluish-white than varying the DC level of a stiff DC current.

These data altogether are intended to be used as guidelines for drivers, concerning the output current fed to the LEDs. Future works should address also the effect of such driving techniques in LED lifespan, and should be carried taking into account a larger number of different devices and models.

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