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Introduction to Variable Inductor Operation and Applications in Power Electronics

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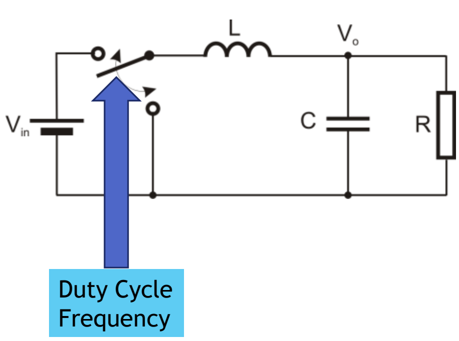
Outline

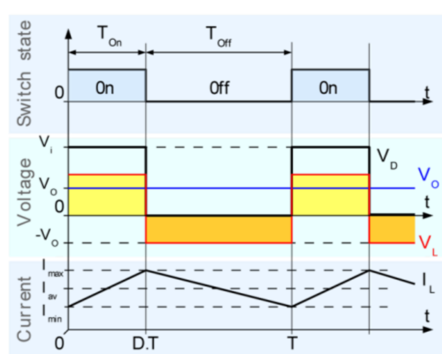
- Introduction to VIs: Why? What? How?
- Analytical Modeling of VIs
- SPICE-based Modeling of VIs
- Examples of Applications
- Conclusions

Why?

Why use variable inductors?

- Variable inductors can provide additional control parameters to improve converter operation:

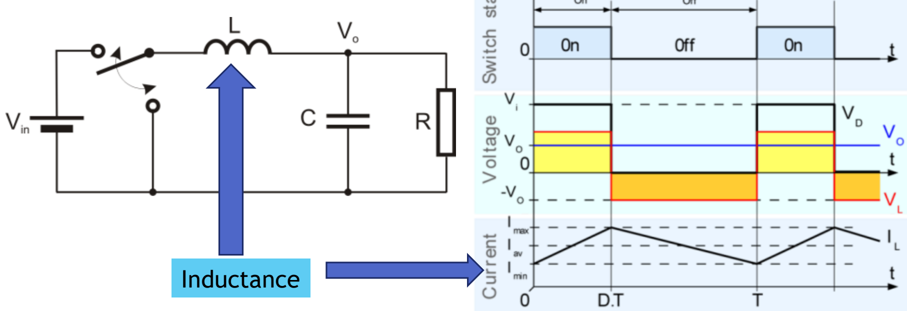




Why use variable inductors?

- Variable inductors can provide additional control parameters to improve converter operation:

BUCK DC-DC CONVERTER



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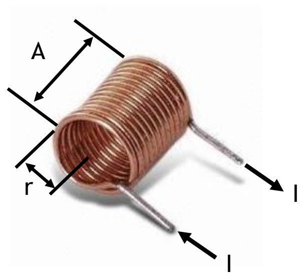
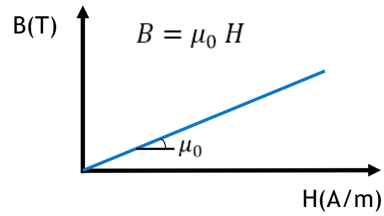
What?

What is a variable inductor?

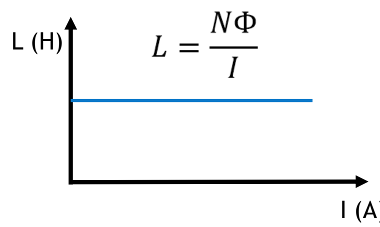
Air Core Inductors are non-variable inductors:





$$L = \frac{0.394\, r^2\, N^2}{9r + 10A}$$

L = inductance (in microhenries)
r = radius of coil (in cm)
N = number of turns
A = length of winding (in cm)



- μ_0=B/H is constant
- The inductance does not depend on the current or any other electrical parameter

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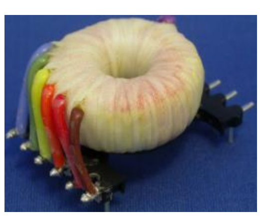
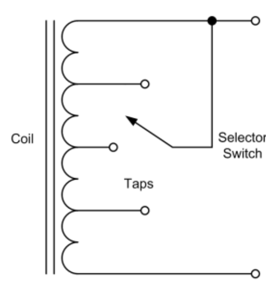
What is a variable inductor?

Tapped inductors can be used as variable inductors:





- Discrete variation of the inductance
- Difficult implementation of selector switch (bidirectional), especially at high frequency
- Difficult implementation of closed-loop control
- Not practical in many applications

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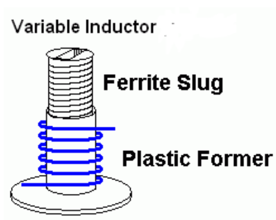
What is a variable inductor?

Variable inductors based on moving cores:



- Continuous variation of the inductance
- Mechanical control of the inductance
- Difficult implementation of closed-loop control
- Not practical in many applications

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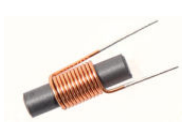
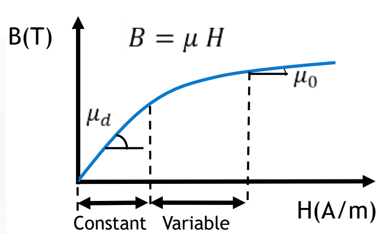
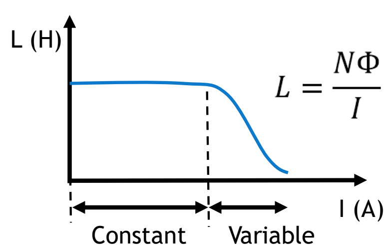
What is a variable inductor?

Magnetic Core Inductors are inherent variable inductors:





- μ is variable, it depends on the operating point (B, H)
- The inductance depends on the current circulating through the winding
- It has no independent control



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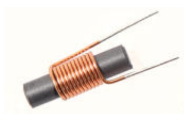


Saturation

≠

Short Circuit

- μ is variable, it depends on the operating point (B, H)
- The inductance depends on the current circulating through the winding
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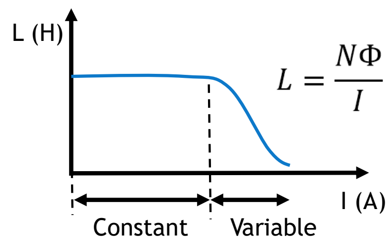

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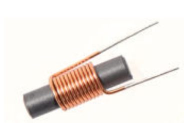
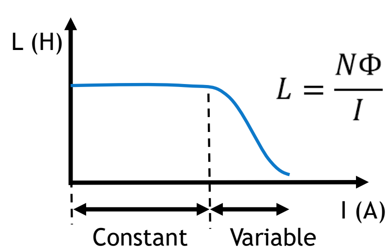
What is a variable inductor?

Magnetic Core Inductors are variable inductors:





- For variable inductor operating in the saturation region the effective inductance must be considered:

$$V = \frac{d(N\Phi)}{dt} = \frac{d(L(i)\cdot i)}{dt} = L(i)\cdot\frac{di}{dt} + i\cdot\frac{dL(i)}{dt} = \left(L(i) + i\cdot\frac{dL(i)}{di}\right)\cdot\frac{di}{dt} = L_{eff}\cdot\frac{di}{dt}$$

$$V = L_{eff}\cdot\frac{di}{dt} \longrightarrow L_{eff} = L(i) + i\cdot\frac{dL(i)}{di}$$

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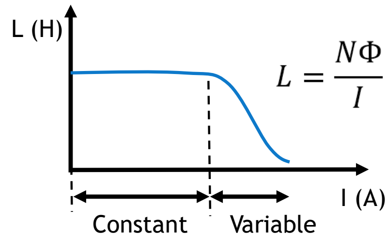
What is a variable inductor?

Magnetic Core Inductors are variable inductors:



- This type of variable inductor can be used in different applications:
  - Power factor correction, Maximum Power Point Tracking in PV applications, voltage regulation, etc.[1]
- However, application is not flexible due to its inherent dependence of the inductance versus current
- A controllable variable inductor would be much more interesting for general applications

[1] W.G. Hurley, W.H. Wölfle; Transformers and Inductors for Power Electronics. Wiley. 2013.

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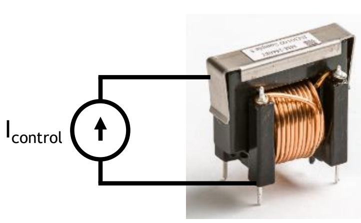
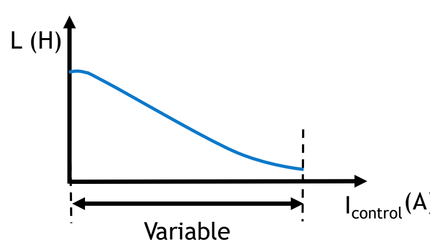
Controllable variable inductor

- A controllable variable inductor is a device whose inductance can be varied by using an electrical magnitude, typically a current:





- Now, the inductance depends on the control current
- The effect of the current circulating through the inductance can be neglected
- The equation that rules the VI operation will be simpler:

$$V = L(I_{control})\frac{di}{dt}$$

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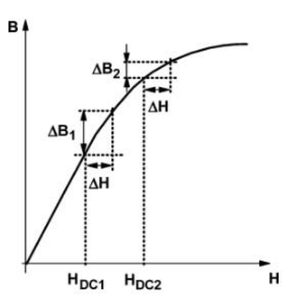
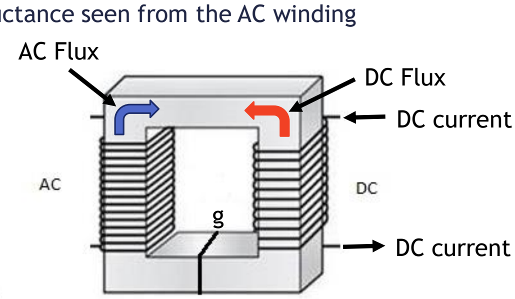
How?

How to make a controllable variable inductor?

- The trick is to superpose a DC flux to the AC flux created by the inductor main winding
- By shifting the DC operating point along the B-H curve the permeability is modulated
- This modifies the inductance seen from the AC winding





$$L_{AC} = \frac{N^2}{R} = \frac{N^2}{\frac{le}{\mu A_e} + \frac{g}{\mu_0 A_e}}$$

Variable Inductance: Depends on DC current

Modulated by DC flux → μA_e

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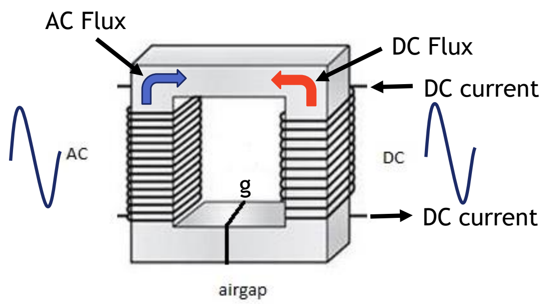
How to make a controllable variable inductor?

PROBLEM:

- The AC flux generates a voltage across the DC winding
- This voltage can be high and could damage the DC current source used to supply the DC winding.



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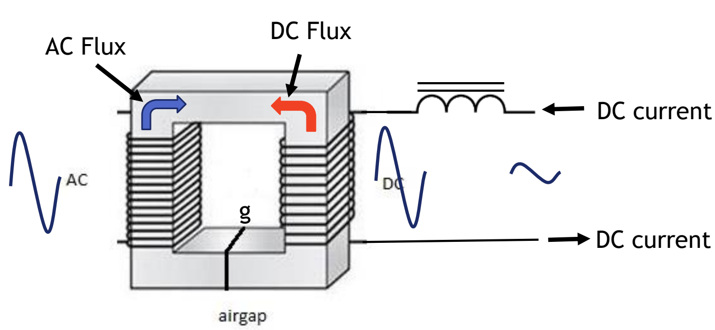
How to make a controllable variable inductor?

A POSSIBLE SOLUTION:

- Adding a series choke to the DC winding can filter the reflected AC voltage



- Disadvantages: complex, bulky and inefficient

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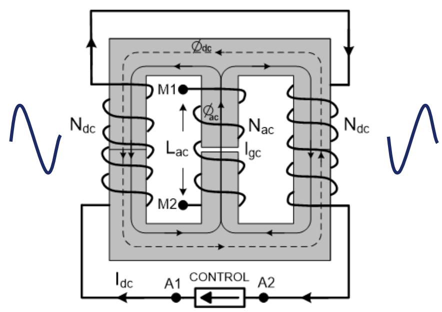
How to make a controllable variable inductor?

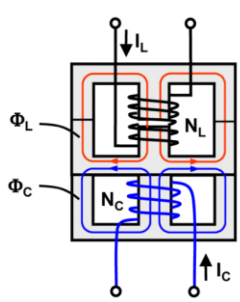
A BETTER SOLUTION:

- Adding two auxiliary windings in reverse polarity on the outer legs
- Both DC windings generate DC flux in the same direction
- The AC flux enters each DC winding with a different polarity
- As a consequence, the AC voltages reflected across the DC windings compensate each other
- The DC current source sees almost no AC voltage across its terminals

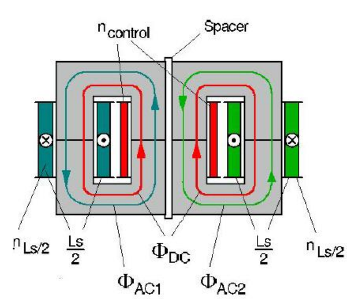


Other VI Structures

Triple E structure



Quadruple U structure



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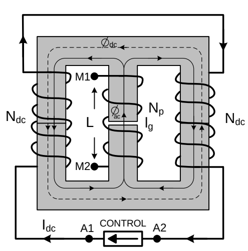
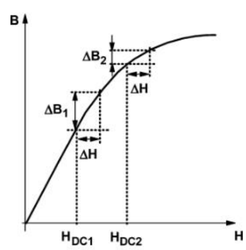
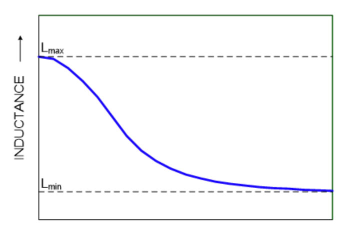
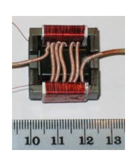
Double-E Variable Inductor







- Main ac winding on the central arm
- DC bias windings on the lateral arms
- DC bias current modulates material permeability



Goal: Obtain the variation of the inductance as a function of the dc bias current. It is also important to determine the total inductance seen from the auxiliary windings because this inductance will strongly affect to the dynamic behavior of the VI.

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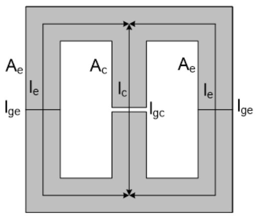
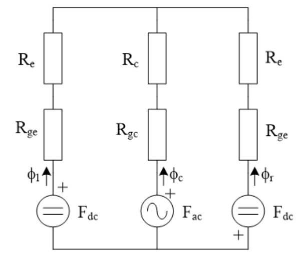
Reluctance Equivalent Circuit





Reluctances	
$\Re_c$	$\dfrac{l_c}{\mu(B)\,A_c}$
$\Re_e$	$\dfrac{l_e}{\mu(B)\,A_e}$
$\Re_{gc}$	$\dfrac{l_{gc}}{\mu_0\,\nu_c\,A_c}$
$\Re_{ge}$	$\dfrac{l_{ge}}{\mu_0\,\nu_e\,A_e}$

MMF	
$\mathcal{F}_{ac}$	$N_{ac}\,I_{ac}$
$\mathcal{F}_{dc}$	$N_{dc}\,I_{dc}$
Magnetic Material	
$H(B) = (k_1 e^{k_2 B^2} + k_3)B$	
$\mu_t(B) = \dfrac{B}{H} = \left[k_1 e^{k_2 B^2} + k_3\right]^{-1}$	
$\mu_d(B) = \dfrac{dB}{dH} = \left[k_1(1+2k_2B^2)e^{k_2 B^2} + k_3\right]^{-1}$	

- It is assumed that the ac component is small compared to the dc level
- Superposition law can be applied to calculate dc and ac components

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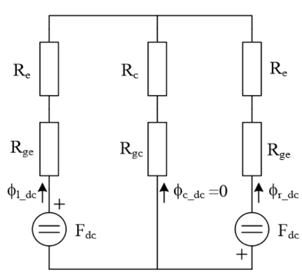
DC Equivalent Circuit



- Analyzing the circuit:

$$\phi_{c\_dc} = 0 \qquad \phi_{l\_dc} = -\,\phi_{r\_dc} = \frac{\mathcal{F}_{dc}}{\Re_e + \Re_{ge}}$$

$$\phi_{l\_dc} = B_{dc}A_e = \frac{\mathcal{F}_{dc}}{\Re_e + \Re_{ge}} = \frac{N_{dc}\,I_{dc}}{\dfrac{l_e}{\mu_t(B_{dc})A_e} + \dfrac{l_{ge}}{\mu_0 \nu_e A_e}}$$

- The following equation must be solved to obtain the magnetic flux density dc level:

$$B_{dc} - \frac{N_{dc}I_{dc}}{l_e\left(k_1 e^{k_2 B_{dc}^2} + k_3\right) + \dfrac{l_{ge}}{\mu_0 \nu_e}} = 0$$

- Once solved, the dc winding inductance can be calculated:

$$L_{dc} = 2\frac{N_{dc}\emptyset_{dc}}{I_{dc}} = \frac{2\,N_{dc}\,B_{dc}(I_{dc})\,A_e}{I_{dc}}$$

$$L_{dc\_eff} = L_{dc}(i_{dc}) + I_{dc}\frac{dL_{dc}(i_{dc})}{di_{dc}} \qquad v_{L\_dc} = L_{dc\_eff}\frac{di_{dc}}{dt}$$

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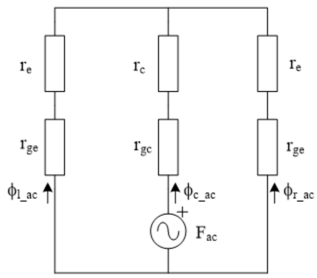
AC Equivalent Circuit



- AC reluctances must be considered:

$$r_e = \frac{l_e}{\mu_d(B_{dc})A_e} \qquad r_{gc} = \frac{l_{gc}}{\mu_0 \nu_c A_e}$$

$$r_c = \frac{l_c}{\mu_d(0)A_c} \qquad r_{ge} = \frac{l_{ge}}{\mu_0 \nu_e A_e}$$

$$\phi_{l\_ac}(B_{dc}) = \phi_{r\_ac}(B_{dc}) = -\phi_{c\_ac}(B_{dc})/2$$

- Calculation of AC flux and AC inductance is straightforward:

$$\phi_{c\_ac}(B_{dc}) = \frac{N_{ac}\, I_{ac}}{r_c + r_{gc} + \frac{r_e + r_{ge}}{2}} = \frac{N_{ac}\, I_{ac}}{\frac{l_c}{\mu_d(0)\, A_c} + \frac{l_g}{\mu_0\, \nu_c\, A_c} + \frac{l_e}{2\, \mu_d(B_{dc})\, A_e} + \frac{l_{ge}}{\mu_0\, \nu_e\, A_e}}$$

$$L_{ac} = \frac{N_{ac}\, \phi_{c\_ac}}{I_{ac}}$$

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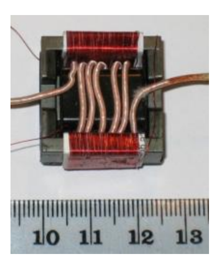
Analysis Example

TABLE II. VARIABLE INDUCTOR DATA

Variable Inductor	
AC winding turns, N_{ac}	6
DC winding turns, N_{dc}	65
DC control current, I_{dc}	0 – 0.5 A
AC current range, I_{ac}	0 – 6 A
Central arm airgap length, l_{gc}	0.6 mm
Estimated central fringing factor, ν	1.06
Lateral arms airgap length, l_{ge}	0.03 mm
Estimated external fringing factor, ν_e	1.0
Expected inductance range, L_{ac}	1.5 – 4.5 μH
EFD25/13/9 (TDK)	
Outer arms length, l_e	43.6 mm
Outer arms area, A_e	28.7 mm²
Center arm length, l_c	24.4 mm
Center arm area, A_c	59.3 mm²
N87 (TDK)	
Initial permeability, 25ºC, μ_i	2200
Flux density at H=1200 A/m, 25ºC	490 mT
Optimum frequency range	25 kHz – 500 kHz
Brauer's model parameters [21]	$k_1 = 0.062\ Am^{-1}T^{-1}$ $k_2 = 42.995\ T^{-2}$ $k_3 = 302.904\ Am^{-1}T^{-1}$



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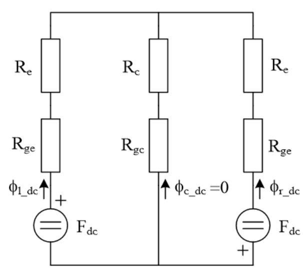
Analysis Example

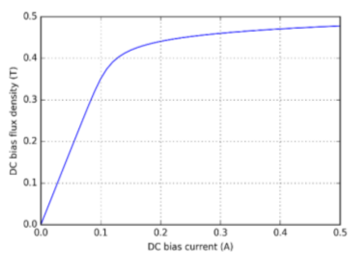


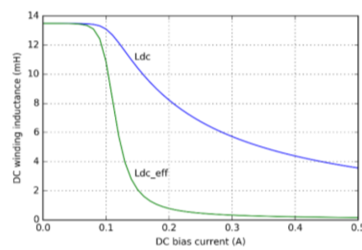
- DC magnetic flux density:

$$B_{dc} - \frac{N_{dc} I_{dc}}{l_e\left(k_1 e^{k_2 B_{dc}^2} + k_3\right) + \frac{l_{ge}}{\mu_0 \nu_e}} = 0$$

- DC winding inductance:

$$L_{dc} = 2\frac{N_{dc}\emptyset_{dc}}{I_{dc}} = \frac{2\, N_{dc}\, B_{dc}(I_{dc})\, A_e}{I_{dc}} \qquad L_{dc\_eff} = L_{dc}(i_{dc}) + I_{dc}\frac{dL_{dc}(i_{dc})}{di_{dc}}$$





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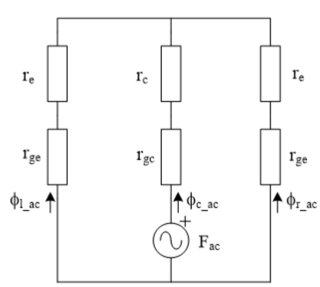
Analysis Example

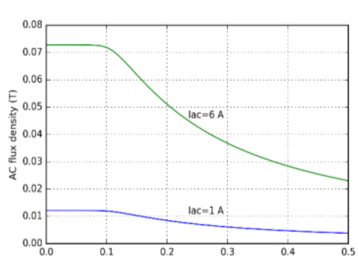


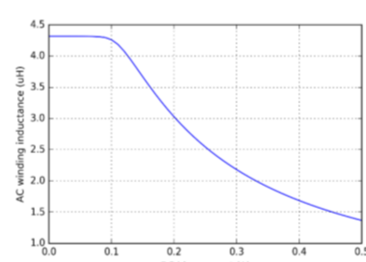
- AC magnetic flux:

$$\phi_{c\_ac}(B_{dc}) = \frac{N_{ac}\, I_{ac}}{r_c + r_{gc} + \frac{r_e + r_{ge}}{2}} = \frac{N_{ac}\, I_{ac}}{\frac{l_c}{\mu_d(0)\, A_c} + \frac{l_g}{\mu_0\, \nu_c\, A_c} + \frac{l_e}{2\, \mu_d(B_{dc})\, A_e} + \frac{l_{ge}}{\mu_0\, \nu_e\, A_e}}$$

- AC winding inductance:

$$L_{ac} = \frac{N_{ac}\, \phi_{c\_ac}}{I_{ac}}$$





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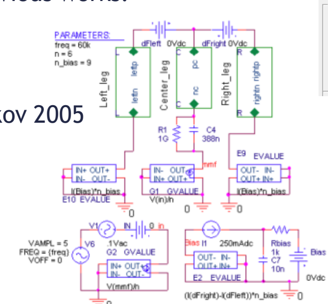
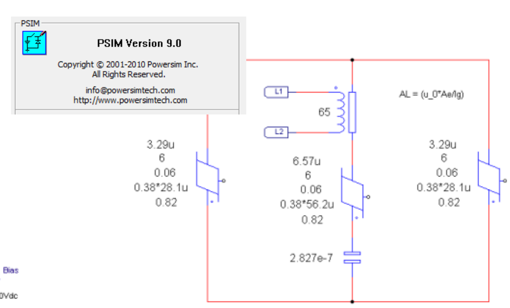
SPICE Modeling of VI

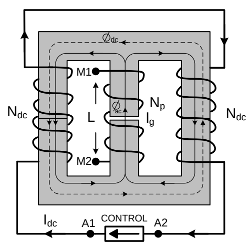
- Importance of modeling of magnetic devices prior to manufacturing
- Provide a better understanding of complex magnetic structures
- Two typical modeling methods:
  - Finite Element Analysis (FEA): high accuracy, time consuming.
  - SPICE based behavioral models: quicker implementation and simulation.
- Previous works:

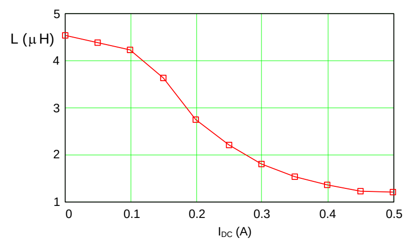
Ben-Yaakov 2005

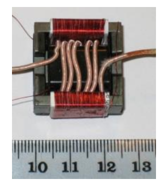




Example of Application: Variable Inductor

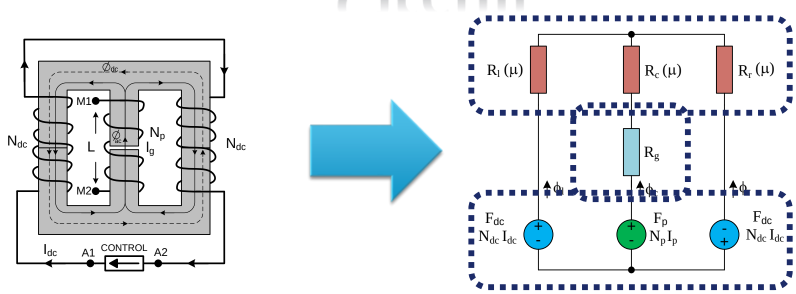






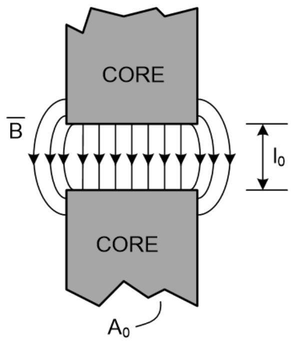
- Relatively complex structures
- Several windings in different arms
- AC and DC excitation
- Different applications

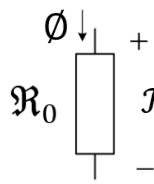
Reluctance Equivalent Circuit



- Required elements:
  - **Constant reluctance elements**: air gap, non magnetic materials
  - **Variable reluctance elements:** depend on magnetic flux density (B)
  - **Windings:** magnetic - electrical interaction

Constant Reluctance Element: Airgap





$$\mathcal{F} = \emptyset \cdot \Re_0$$

$$\Re_0 = \frac{l_0}{\mu_0 A_0 \nu}$$

l_0 – *length*

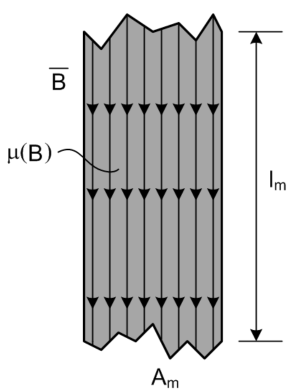
A_0 – *area*

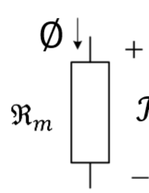
ν – *fringing factor*

μ_0 – *permeability of free space*

- In SPICE simulator:
  - Voltage ⟷ Magnetomotive Force
  - Current ⟷ Magnetic Flux
  - Resistance ⟷ Reluctance

Variable Reluctance Element



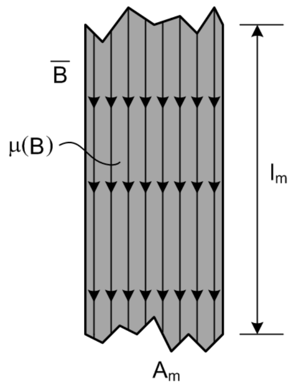


$$\mathcal{F} = \emptyset \cdot \Re_m$$

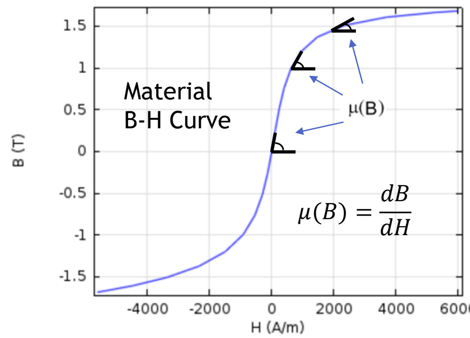
$$\Re_m = \frac{l_m}{\mu \; A_m}$$

- But $\Re_m$ can only be considered as a constant when the material operates in linear region!

Variable Reluctance Element



- In a general case the material has to be modeled in the whole range:



- Reluctance depends on Magnetic Flux density B: $\Re_m(B) = \dfrac{l_m}{\mu(B)\ A_m}$

- Next step is to model the B-H Curve and to obtain a valid expression for $\mu(B)$

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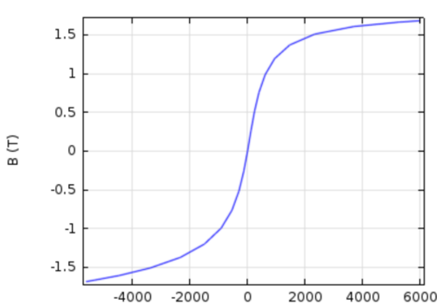
B-H Curve Modeling

- Neglecting hysteresis effects
- Simple relationship to avoid SPICE convergence problems



- Hyperbolic model:

$$H(B) = \frac{k_1}{\frac{k_2}{B} + 1}$$

- Brauer's model:

$$H(B) = \left(k_1 e^{k_2 B^2} + k_3\right) B$$

$$\frac{d}{dB}$$

- Direct expression of non-linear material permeability: $\mu(B) = \left[k_1(1 + 2k_2 B^2)e^{k_2 B^2} + k_3\right]^{-1}$

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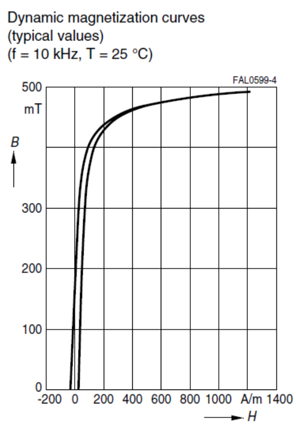
B-H Curve Modeling

- TDK-EPCOS N87 MATERIAl



N87 Datasheet

Ddatasheet =

	0 (Bd)	1 (Hd)	2 (Bi)	3 (Hi)
0	0	24.24	0	-30.3
1	0.1	30.3	0.1	-12.12
2	0.2	48.48	0.2	6.06
3	0.3	72.72	0.3	30.3
4	0.4	133.33	0.4	90.9
5	0.428	200	0.437	200
6	0.46	400	0.464	400
7	0.474	600	0.474	600
8	0.482	800	0.483	800
9	0.488	$1\cdot10^3$	0.489	$1\cdot10^3$
10	0.491	$1.2\cdot10^3$	0.491	$1.2\cdot10^3$

B-H Curve Modeling

- TDK-EPCOS N87 MATERIAl

$$H(B) = \left(k_1 e^{k_2 B^2} + k_3\right) B$$

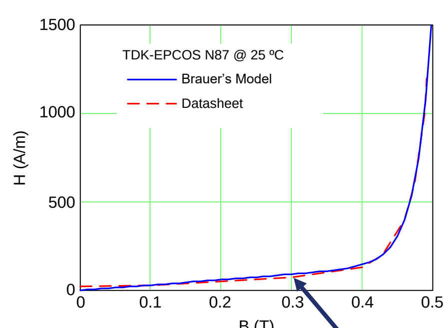


TABLE I. BRAUER'S COEFFICIENTS FOR N87 MATERIAL AT 25ºC

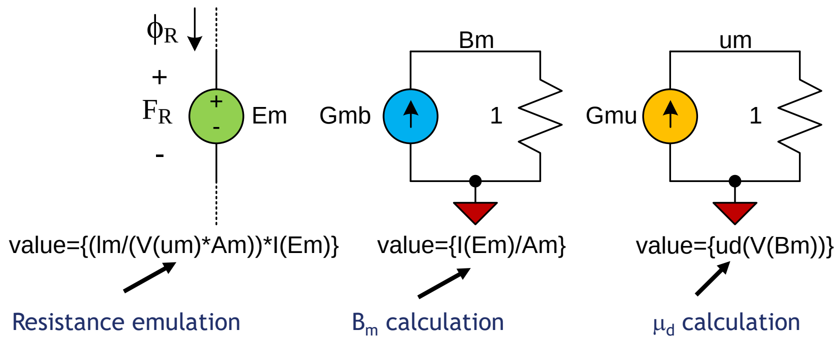
Parameter	Value
k_1	0.062 $Am^{-1}T^{-1}$
k_2	42.995 T^{-2}
k_3	302.904 $Am^{-1}T^{-1}$

$$\mu(B) = \left[k_1(1 + 2k_2B^2)e^{k_2B^2} + k_3\right]^{-1}$$

- Maximum relative error ≈ 26% @ 0.3 T

Variable Reluctance Element
SPICE Implementation

.func ud(B) { 1/((1+2*k2*B*B)*exp(k2*B*B) + k3) } ; differential permeability



- Can be implemented as an isolated component

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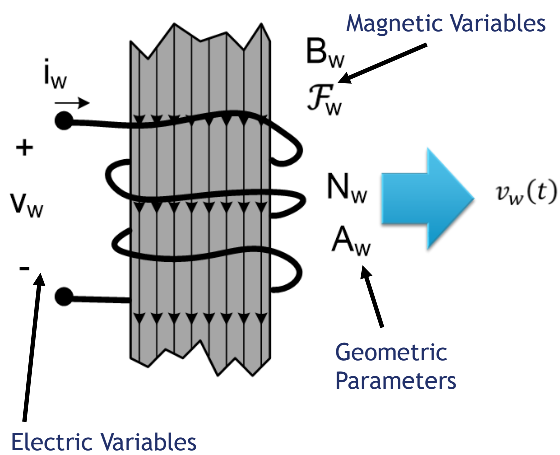
Winding Modelling



$$v_w(t) = N_w \cdot \frac{d\phi_w(t)}{dt} = N_w A_w \frac{dB_w(t)}{dt}$$

$$\mathcal{F}_w(t) = N_w \cdot i_w(t)$$

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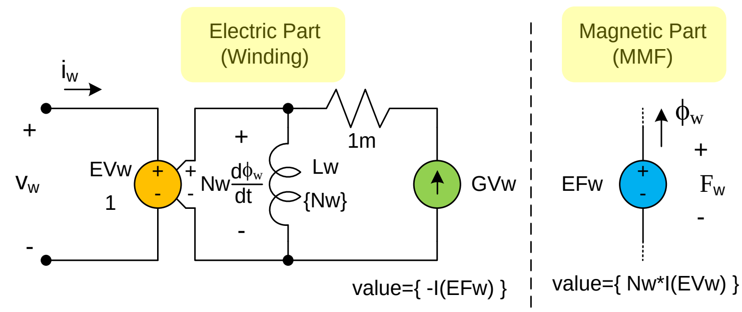
Winding Modelling
SPICE Implementation



Implementation electric-magnetic interaction

$$v_w(t) = N_w \cdot \frac{d\emptyset_w(t)}{dt} = N_w A_w \frac{dB_w(t)}{dt}$$

Implementation of M.M.F.

$$\mathcal{F}_w(t) = N_w \cdot i_w(t)$$

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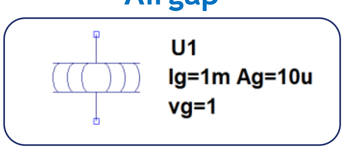
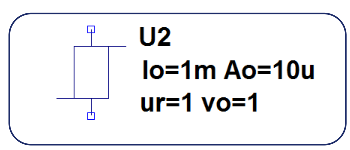
Summary of Basic Components

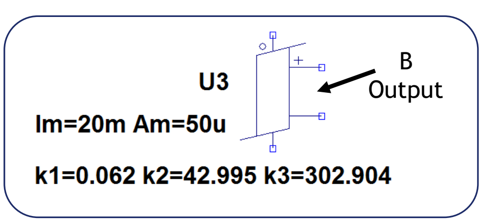
Airgap



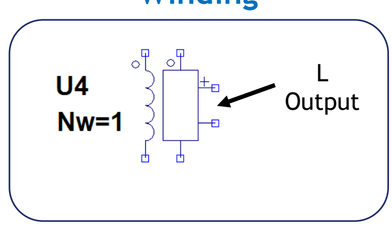
Linear Reluctor



Non Linear Reluctor



Winding



Available for evaluation at: http://www.unioviedo.net/ate/marcosaa

Example: Variable Inductor

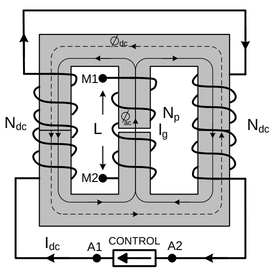
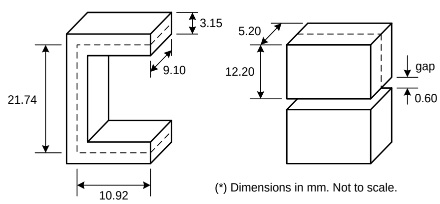
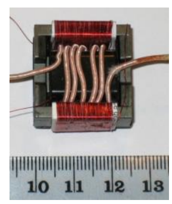


TABLE II. VARIABLE INDUCTOR DATA

Core material	N87 TDK-EPCOS	Air gap length	0.6 mm
Core type	EFD25	Estimated fringing factor (ν)	1.06
Expected inductance range	1.2 µH – 4.5 µH	Central arm average length (lc)	24.4 mm
Main winding turns (N_p)	6	Central arm section (A_c)	59.3 mm^2
Intended operation regime	Sinusoidal, 500 kHz	Outer arms average length (l_1)	43.6 mm
Main winding peak current	6.0 A	Outer arms section (A_1)	28.7 mm^2
Bias windings turns (N_{dc})	65		





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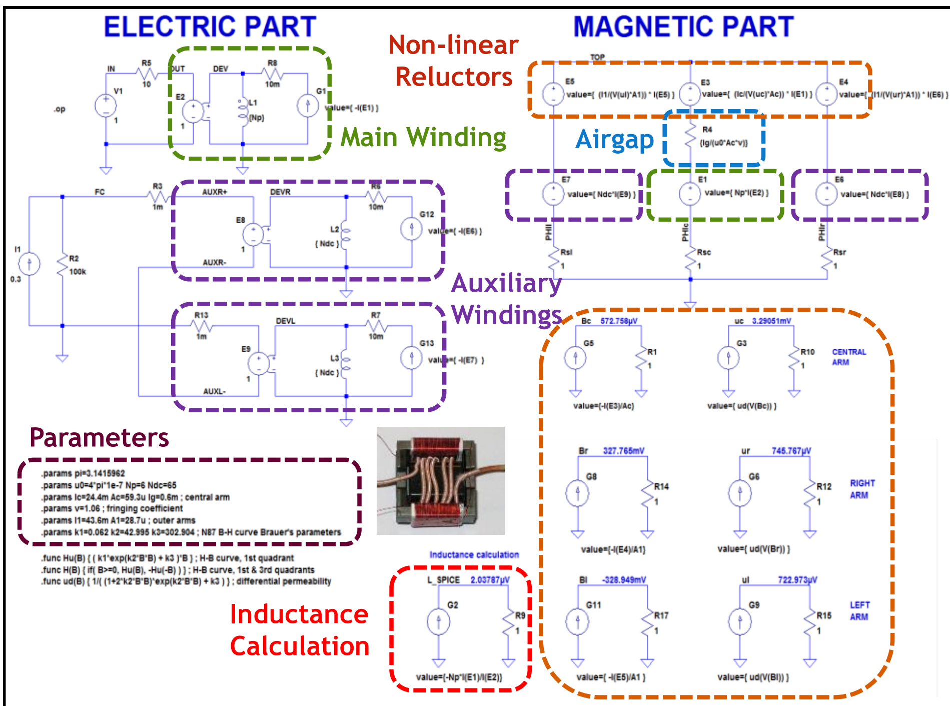

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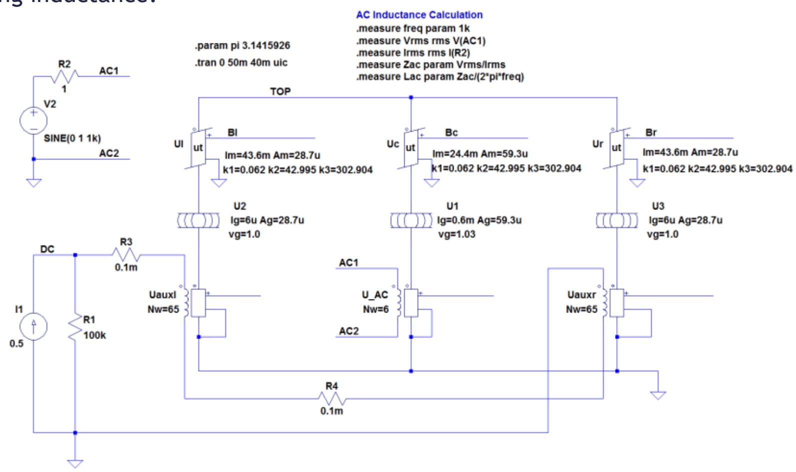
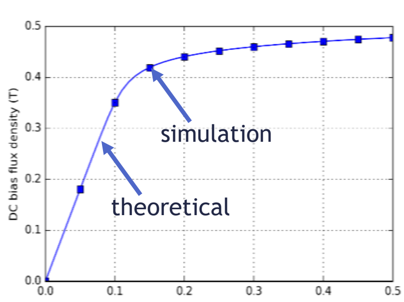
Simulation Results

LTspice circuit for the simulation of the VI, including the calculation of the ac winding inductance:

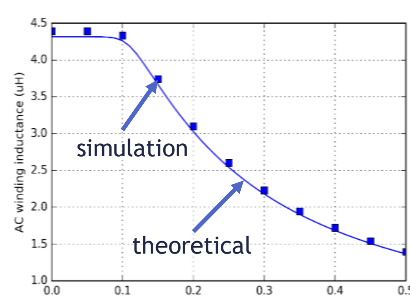


Simulation Results

DC magnetic flux density in the outer arms: theoretical results (blue solid line) and simulation results (blue squares).

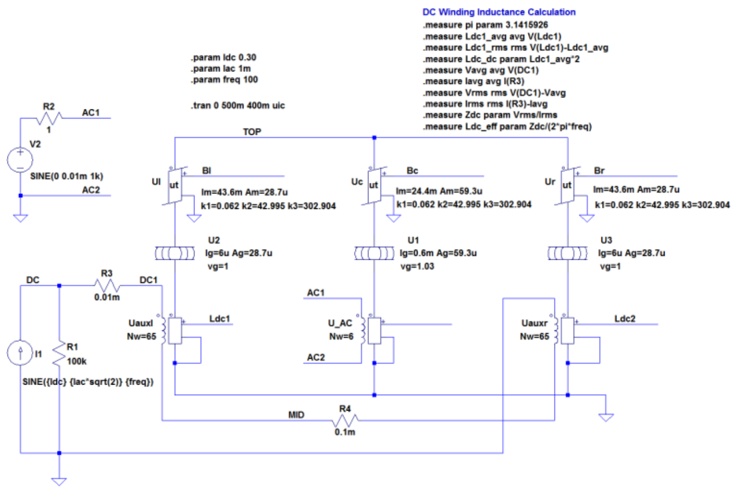


AC winding inductance: theoretical results (blue solid line) and simulation results (blue squares).



Simulation Results

LTspice circuit for the simulation of the VI including the calculation of the dc bias winding inductance.



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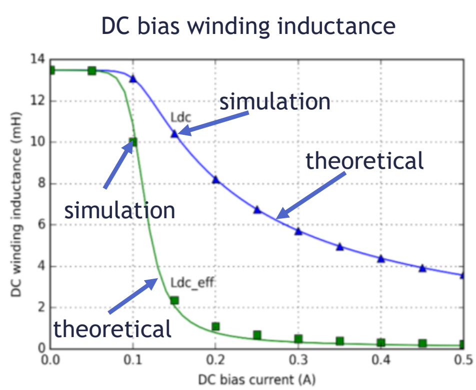
Simulation Results



DC bias winding inductance theoretical results (blue solid) and simulation results (red triangles).

DC bias winding effective inductance (L_(dc_eff)): theoretical results (green solid) and simulation results (blue squares)

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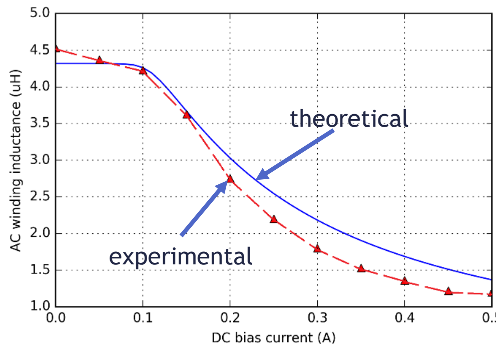
Experimental Results

AC winding inductance



AC winding inductance: theoretical results (blue solid line) and experimental measurements (red triangles).

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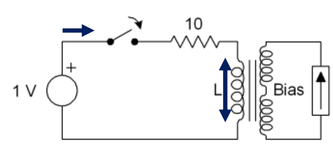
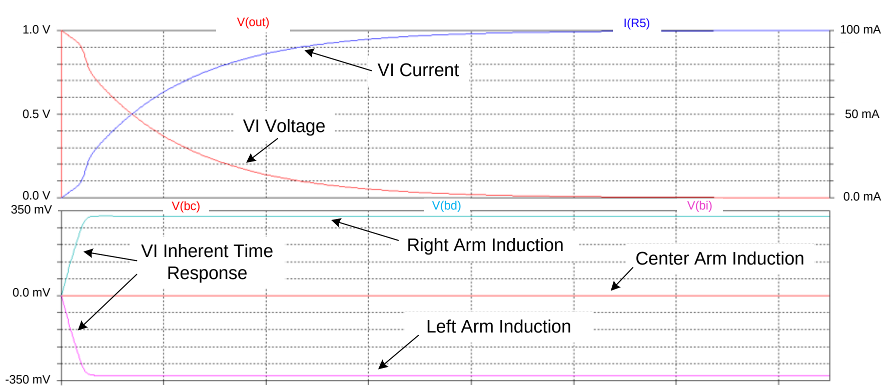
DC Transient Simulation





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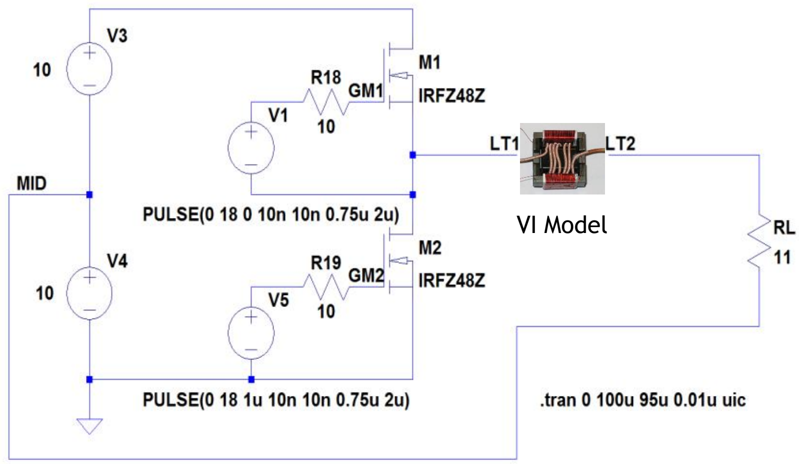
Experimental Results

LR inverter at 500 kHz



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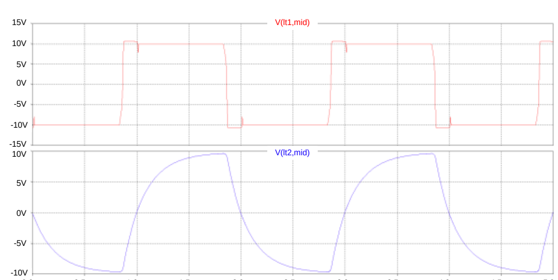
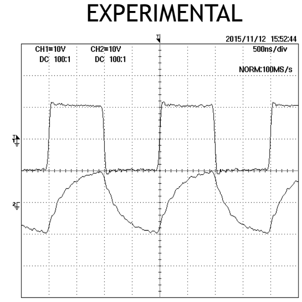
Experimental Results

LR inverter at 500 kHz

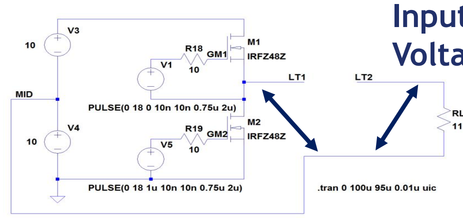
SIMULATION



EXPERIMENTAL



Input and Output Voltage



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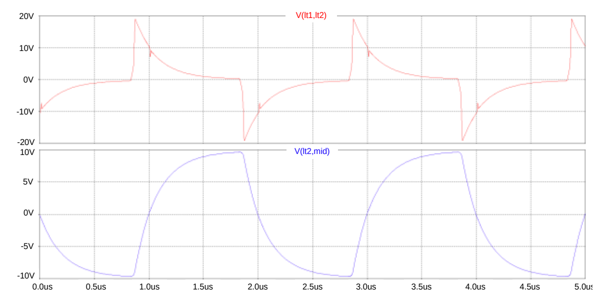
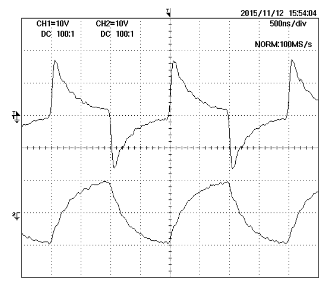
Experimental Results
LR inverter at 500 kHz

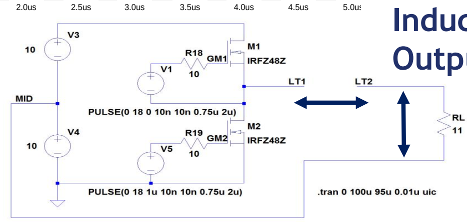
SIMULATION

EXPERIMENTAL





Inductor and Output Voltage



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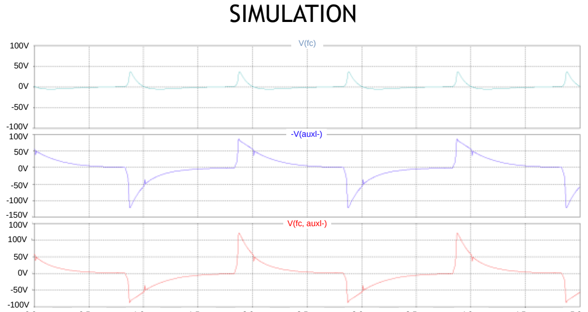
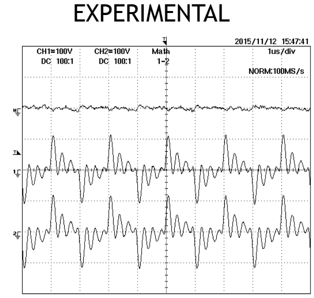
Experimental Results
LR inverter at 500 kHz

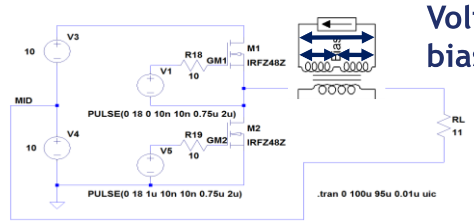
SIMULATION

EXPERIMENTAL





Voltage across bias windings



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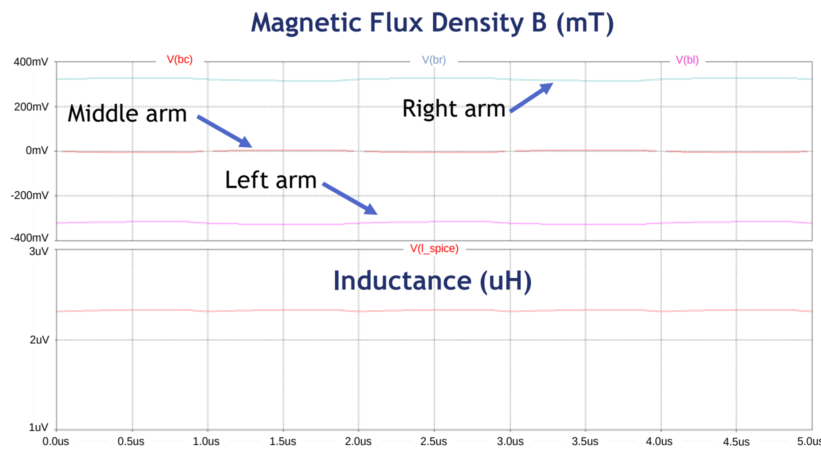
Simulation Results
LR inverter at 500 kHz



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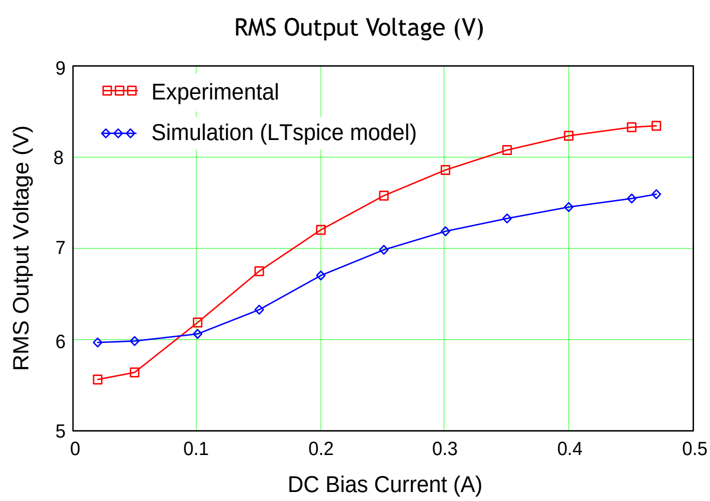
Simulation Results
LR inverter at 500 kHz



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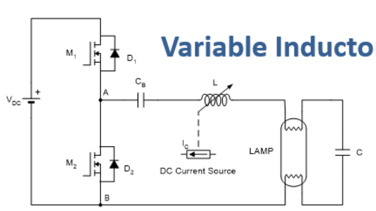
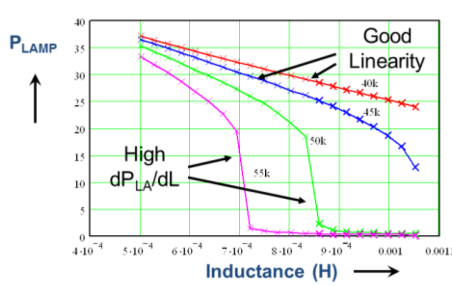
Outline

- Introduction to VIs
- Analytical Modeling of VIs
- SPICE-based Modeling of VIs
- Examples of Applications
- Conclusions

Control of Resonant Inverters

Use of Magnetic Regulators to Improve Power Converter Performance

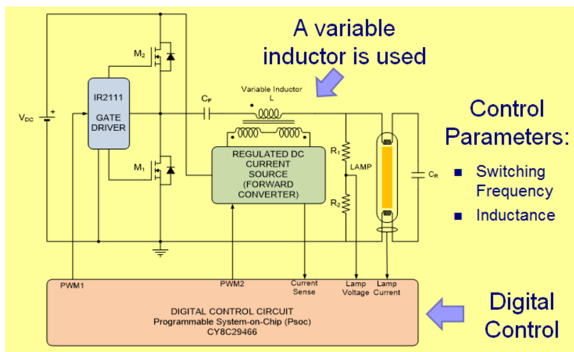




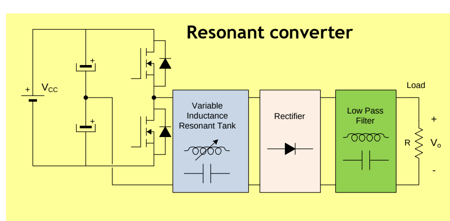
$$P_{LA} = \frac{V_{LA}^2}{R_{LA}} = \frac{V_E^2 R_{LA}}{R_{LA}{}^2 \cdot \left(1 - 4\pi^2 f^2 LC\right)^2 + 4\pi^2 f^2 L^2}$$

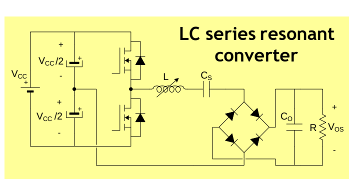
Universal Ballasts

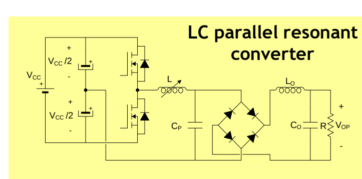
Universal Ballast with Variable Inductance and Digital Control



Control of DC-DC Resonant Converters

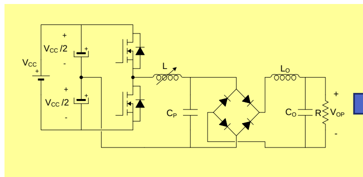




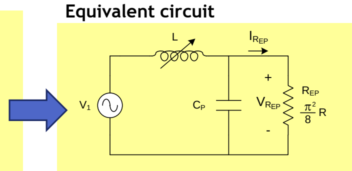


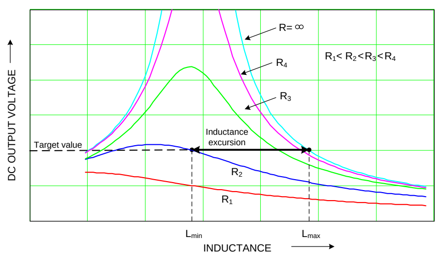
Control of Resonant Inverters

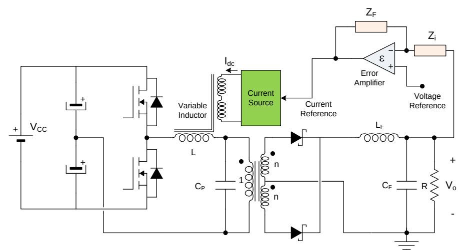
Magnetic Control of PRC



Equivalent circuit





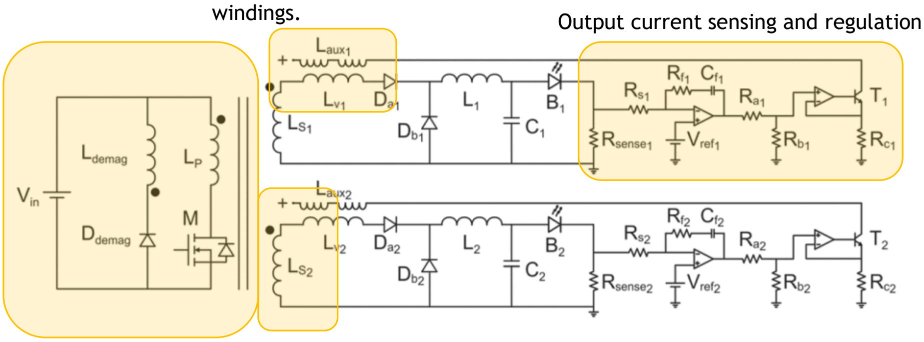


Can be used to control other resonant converters at constant frequency

Current Equalization in LED Drivers

Variable inductor w/ auxiliary windings.

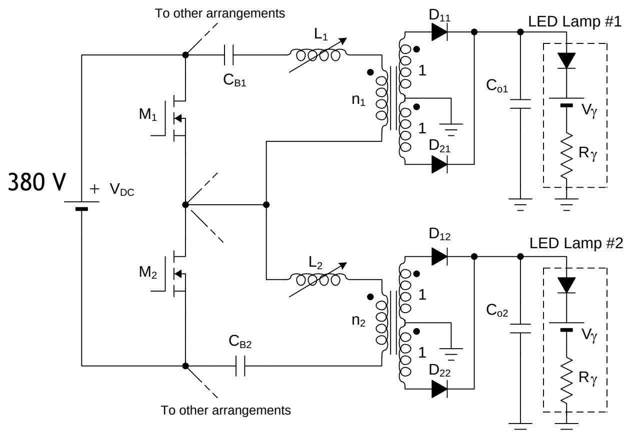
Output current sensing and regulation



Forward converter with demag. winding.

One output for each LED branch.

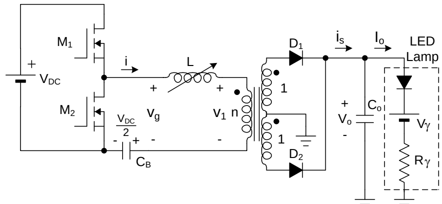
Single Inductor LED Driver

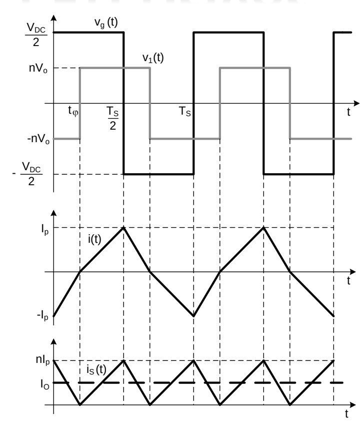


Multi-array LED lamp

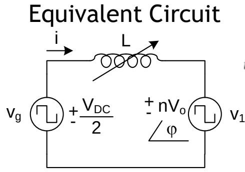
- Application to several LED arrays
- Each branch can be controlled independently

Single Inductor LED Driver





Equivalent Circuit

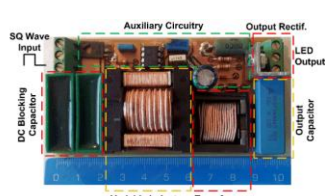


$$i(t) = -I_p + \frac{0.5V_{DC} + nV_o}{L}t \quad t \in [0, t_\varphi]$$

$$i(T_s/2) = -i(0)$$

$$i(t) = \frac{0.5V_{DC} - nV_o}{L}(t - t_\varphi) \quad t \in [t_\varphi, T_s/2]$$

$$I_o = \frac{nV_{DC}^2 - 4n^3V_O^2}{16V_{DC}Lf_s}$$



Summary and Conclusions

- Variable inductors can be used to provide additional control parameters in power electronics converters.
- Analytical modeling of VI is useful for the first design and evaluation of the variable inductor.
- SPICE based models can be used to simulate the complete converter under VI control.
- VI have been tested successfully to perform control of power converters in different applications
- New ideas and applications are expected in the near future. There are possibilities to develop new ideas using VI for the control of power converters.

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Thank you! Questions?

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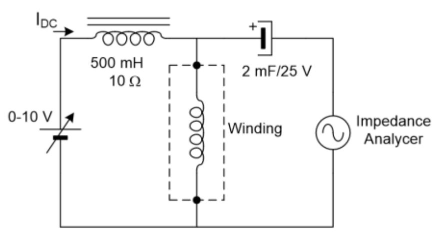
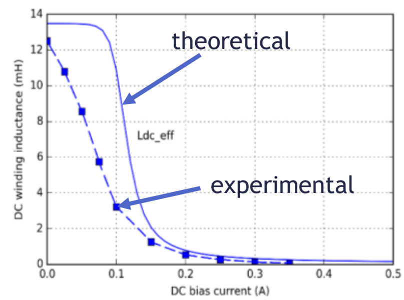
Experimental Results

Effective DC winding inductance: theoretical results (blue solid line) and experimental measurements (blue squares).

Measurement Circuit



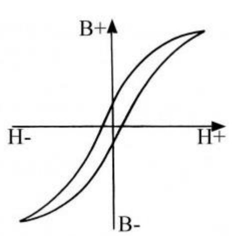
DC Winding Effective Inductance

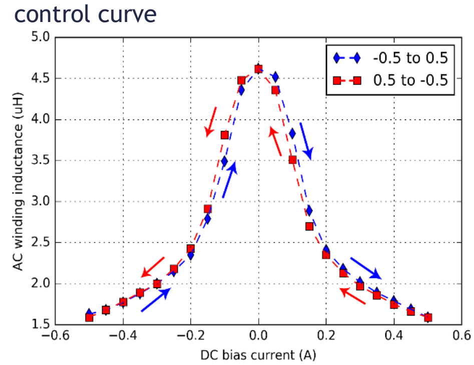


- The approximation is not as good as expected.
- Could be due to the fact that the ac component does not meet the condition of small signal operation on the bias winding. Since the bias magnetic path has a negligible airgap, a small excursion of the magnetic field intensity yields a great excursion of the magnetic field density.

Experimental Results

Effect of the hysteresis behavior of the material on the AC winding inductance control curve





- The ac inductance was measured for one complete hysteretic cycle, starting from a dc current of -0.5A, going up to 0.5 A and back to -0.5 A again.
- There is a small effect of the hysteresis on the control curve, but it is not very relevant.